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Evaluating the Methods of Estimating Total Hours Actually Worked: Insights from Labor Market Statistics

Ocena metod estymacji czasu pracy – wnioski z analizy statystyk rynku pracy

Abstract

This study evaluates methods for estimating total quarterly hours actually worked within enterprises, using known annual values obtained from a complete enumeration survey conducted by the Statistics Poland. Using data from Q1 2009 to Q4 2023, we compare the estimates across economic activity sections with official quarterly survey data from Statistics Poland. Our approach prioritises practicality, computational feasibility, and statistical integrity. The evaluated methods are classified using forecast accuracy metrics and taxonomic tools based on the distance from an abstract ideal solution. The analysis demonstrates that methods employing average paid employment as an auxiliary series are more effective than approaches focused on movement preservation. Litterman's method, which minimises the weighted residual sum of squares, exhibits the highest forecast accuracy and the greatest resilience to external shocks such as the COVID-19 pandemic and the global energy crisis. Our findings provide useful insights for selecting optimal interpolation methods in labour market statistics from a complete enumeration survey by the Statistics Poland.

Keywords:
official statistics, Litterman method, Polish Taxonomic Thought, time actually worked, disaggregation of annual working hours into quarters

JEL classification codes:
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Streszczenie

W badaniu oceniono metody szacowania łącznej kwartalnej liczby godzin przepracowanych w przedsiębiorstwach, wykorzystując do tego znane wartości roczne z pełnego badania przeprowadzanego przez Główny Urząd Statystyczny. Oszacowania dotyczące okresu od I kwartału 2009 r. do IV kwartału 2023 r. uzyskane w poszczególnych sekcjach działalności gospodarczej porównano z oficjalnymi danymi pochodzącymi z badań kwartalnych Głównego Urzędu Statystycznego. W analizie uwzględniono przede wszystkim praktyczność, wykonalność obliczeniową oraz spójność statystyczną. Metody zostały sklasyfikowane na podstawie miar dokładności prognozowania oraz narzędzi taksonomicznych opartych na odległości od abstrakcyjnego idealnego rozwiązania. Wyniki wskazują, że metody, w których wykorzystuje się przeciętną liczbę zatrudnionych jako zmienną pomocniczą, są bardziej efektywne niż te, które koncentrują się na zachowaniu dynamiki zmian. Metoda Littermana, minimalizująca ważoną sumę kwadratów reszt, wykazała najwyższą dokładność prognoz oraz odporność na zewnętrzne

Słowa kluczowe:
statystyka publiczna, metoda Littermana, polska myśl taksonomiczna, czas faktycznie przepracowany, dezagregacja rocznego czasu pracy na kwartały

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szoki, takie jak pandemia COVID-19 i globalny kryzys energetyczny. Uzyskane wyniki dostarczają wskazówek dotyczących wyboru optymalnych metod interpolacji w statystyce rynku pracy.

Introduction

The challenge of missing high-frequency data is a common issue for researchers and practitioners [**Chamberlin, 2010**]. Although no tools can fully compensate for this limitation, temporal disaggregation methods have been widely applied in public statistics, particularly in the context of national accounts [**Mosley et al., 2022**; **Wójcik, 2016**; **Quenneville et al., 2013**; **Chen, 2007**]. In contrast, this article analyses temporal disaggregation from the perspective of practitioners involved in the production of official labour market statistics. The methods considered here prioritise practicality, computational viability, resilience, ease of understanding, statistical integrity, and adaptability [**Quilis, 2018**].

The goal of this article is to disaggregate annual time series of time actually worked (effective working time) in medium-sized and large enterprises into quarterly frequencies across the NACE Rev. 2 economic activity sections. The study evaluates sparse and adaptive sparse temporal disaggregation [**Mosley et al., 2022**], the temporal disaggregation methods of **Sax and Steiner** [**2013**], and the imputation methods of **Moritz and Bartz-Beielstein** [**2017**] from the first quarter of 2009 to the fourth quarter of 2023. It compares the ability of these interpolated time series to mirror the official, high-quality, but costly, complete enumeration survey-based quarterly series on hours actually worked provided by the Statistics Poland (GUS). The assessment is conducted using forecast accuracy metrics, statistical tests, diverse auxiliary time series, and taxonomic analysis based on proximity to an abstract ideal solution [**Hellwig, 1968**; **Walesiak, 2017**].

This article contributes to both the literature and practice as an optimal method depends critically on the application context [**Reber, Pack, 2014**]. Therefore, it is essential to consider the specific nature of effective working time and the availability of suitable high-frequency auxiliary indicators.

The study introduces the first application of **Hellwig's** [**1968**] procedure and Polish taxonomic principles for ranking interpolation methods. Unlike most empirical studies on temporal disaggregation, which usually focus on a single method or compare only a few, this study evaluates 36 methods using a real-world case study. It considers four fundamental high-frequency auxiliary variables – the industrial production index, GDP, GDP *per capita*, and gross value added (all expressed in real terms) – as well as their combinations with average paid employment, including lagged variants and seasonal components. In total, this results in over 36 potential high-frequency auxiliary series across each economic activity section (see Table 5A in the Appendix).

The article is applicative in nature, emphasising the need for sustainable statistics to replace surveys with alternative methods or data sources while maintaining high precision. Time actually worked in enterprises is typically not available from administrative data sources, unlike employment and wage statistics. This necessitates estimating or surveying its missing higher frequency data. Up-to-date quarterly information on time actually worked is crucial in modern knowledge-based economies. Mathematical methods can meet this need, reducing the statistical burden on enterprises, lowering the costs associated with survey-centric approaches, and improving government budget conditions. Enterprises could then allocate their time more productively instead of completing time-consuming quarterly reports.

Empirical findings reveal that, among the methods considered, temporal distribution methods are the most effective for interpolating missing quarterly data on time actually worked within enterprises. Specifically, the results suggest using the average paid employment indicator as an auxiliary time series rather than methods focused primarily on movement preservation. In our application, the **Litterman** [**1983**] method, with the minimisation of the weighted residual sum of squares [**Barbone et al., 1981**], provided the best forecast accuracy. Using only the average paid employment indicator yielded satisfactory results while maintaining simplicity, ease of understanding and consistency across economic activity sections (MAPE = 2.8%). Incorporating additional auxiliary indicators further improved forecasts (MAPE decreased to 2.49%), though the optimal auxiliary indicators varied across economic activity sections.

This article may help overcome scepticism regarding the potential of temporal disaggregation methods for labour market statistics in Poland. The findings offer valuable guidance for selecting the optimal method for temporal disaggregation of effective working time, applicable to labour market statistics in other countries as well.

The remainder of this article is organised as follows. Section 2 provides a review of temporal disaggregation methods, along with their applications in official statistics. Section 3 compares the current approach used by Statistics Poland with the alternative proposed in this study. Section 4 outlines the methodological framework, including the temporal disaggregation procedures and the linear ordering method applied. Section 5 describes the data sources, clarifies key components included in the definition of effective working time, and presents empirical evidence demonstrating a close relationship between average paid employment and time actually worked. Section 6 presents the empirical results. Finally, Section 7 discusses the limitations of the empirical research and proposes future improvements based on administrative data.

Literature review

Temporal disaggregation techniques are regression-based methods employed by several NSOs. For instance, the Italian National Institute of Statistics adopts these techniques to obtain quarterly national accounts [**Bisio, Moauro, 2018**]. In France, temporal disaggregation is utilised to determine the quarterly GDP [**Sax, Steiner, 2013**], while in Austria, numerous quarterly time series are estimated by disaggregating annual data [**Scheiblecker et al., 2007**]. The US Census Bureau applies the **Causey-Trager** [**1981**] growth rate preservation model for temporal disaggregation and benchmarking [**Brown, 2012**]. **Rossi and Chini** [**2021**] utilise a state-space method for the temporal distribution of US business dynamics, deriving quarterly data from the Census data. **Pipień and Roszkowska** [**2015**] propose a linear regression method for estimating Poland's quarterly GDP.

Interpolation methods typically fall into two categories: (1) methods that do not use auxiliary time series and rely solely on mathematical dependencies or ARIMA models to smooth the unobserved values [**Hodgess, Mhoon, 2019**]; and (2) methods that incorporate information from auxiliary time series available at higher frequencies, which are logically and empirically related to the lower frequency series with missing data. Comprehensive overviews of temporal disaggregation, interpolation, benchmarking, and calendarisation techniques are provided by **Dagum and Cholette** [**2006**], **Pavía-Miralles et al.** [**2010**], and the **International Monetary Fund** [**2017**].

Early work in this field includes **Denton's** [**1971**] quadratic optimisation approach focused on movement preservation to minimise the difference between revised and original series. **Chow and Lin** [**1971**] introduced a generalised least squares (GLS) regression using several high-frequency auxiliary variables, including a possible constant, a method now well-established in several NSOs [**Eurostat, 2018**]. This technique has been extended by **Fernández** [**1981**] and **Litterman** [**1983**], who allow residuals to follow a nonstationary process. **Fernández's** [**1981**] approach is a special case of **Litterman's** [**1983**], where residuals follow a random walk. Litterman's method more broadly considers the serial correlation of errors. **Santos Silva and Cardoso** [**2001**] further enhanced the **Chow and Lin** [**1971**] method by incorporating dynamic models.

Recent advancements in big data and the increasing availability of high-frequency administrative data have spurred a growing body of literature on forecasting macroeconomic time series [**Fuleky, 2019**]. High-frequency administrative data enable the use of machine learning methods and numerous high-frequency indicators to generate more accurate forecasts. However, when the number of dimensions exceeds that of observations, consistent parameter estimates require additional structure [**Wainwright, 2019**]. Consequently, recent approaches have extended earlier work on temporal distribution to high-dimensional settings [**Mosley et al., 2022**].

Alternative proposals for temporal distribution include seemingly unrelated structural time series models [**Moauro, Savio, 2005**] and multivariate models with restrictions imposed by time series [**Proietti, 2010**]. **Huang** [**2012**] suggests a regime-switching model to capture the non-linear behaviour of aggregated and disaggregated output series and the asymmetrical nature of business cycles. A novel approach is the use of wavelets for temporal disaggregation [**Perricone, 2018**], which have been gaining popularity in economics [**Ryczkowski, Zinecker, 2022**]. Despite these innovations, NSOs typically use well-established, sound methods recognised and accepted by the international community, as noted by **Di Fonzo** [**2003**]. Moreover, **Quilis** [**2018**] argues that the benefits of increased complexity in temporal distribution methods may be minimal.

The basis for a new estimation of effective working time

By providing a robust and practical solution for estimating quarterly hours worked, this research seeks to reduce reliance on costly and time-consuming survey (Z-03), thereby enhancing the efficiency of official statistics. In the method currently employed by the Statistics Poland for determining quarterly total time actually worked in enterprises, the actual hours worked for Q1, Q2, and Q3 are calculated as the sum of hours reported in Forms Z-03 and DG-1. The annual total time actually worked for the same group of units is determined based on reports submitted using Form Z-06. Administrative data sources play a supporting role in calculating selected specific values related to employment (upper part of Figure 1).

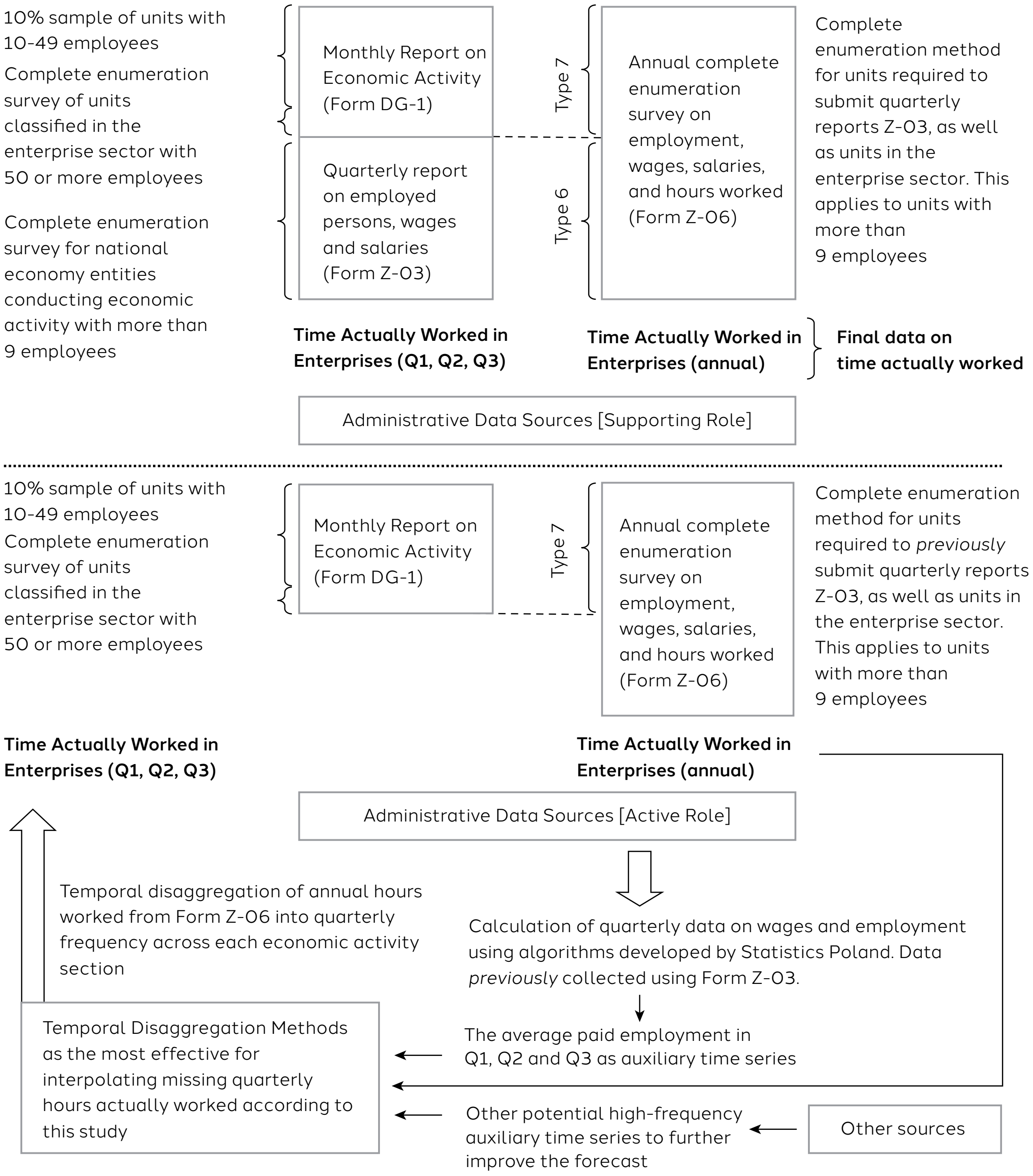
The survey conducted using Form Z-03 provides information on time actually worked by employees considered in the calculation of the average number of employed persons. The Z-03 survey also covers public sector units and budgetary institutions classified under all types of activities, regardless of the number of employees. However, it does not include agricultural holdings, foundations, associations, social organisations, employers' organisations, or economic and professional self-government bodies.

The survey conducted using Form DG-1, similar to the survey with Form Z-03, provides information on time actually worked by employees considered in the calculation of the average number of employed persons. The DG-1 survey covers entities classified within the enterprise sector with 50 or more employees, as well as a 10% sample of units with 10 to 49 employees.

The proposed new procedure for determining quarterly data on hours actually worked is based on the temporal disaggregation of annual data (Form Z-06), combined with quarterly data on average employment from administrative data sources (as an auxiliary variable) and other potential high-frequency auxiliary time series to further improve the forecast. Here, administrative data sources play an active role, as the algorithms for calculating employment, wages, and salaries developed by **Statistics Poland** [**2018**], and later refined, are now ready to generate the quarterly data on wages and employment that were previously collected using Form Z-03 (see the lower part of Figure 1).

The only data missing from administrative sources are hours actually worked, making their estimation crucial for eliminating the need for the quarterly Z-03 survey. A central question addressed in this paper is how the estimated data on time actually worked, from a historical perspective spanning Q1 2009 to Q4 2023, differed from the actual survey-based data on hours worked as obtained from the combined Z-03 and DG-1 sources. To clarify, the Monthly Report on Economic Activity (Form DG-1) would remain necessary even if the method proposed in this article were adopted. This is because the DG-1 survey includes questions about economic activity, many of which are not covered by administrative sources, nor has a methodology been established for estimating them.

Figure 1. Current method of the Statistics Poland for determining quarterly time actually worked (above) and proposal using temporal disaggregation methods (below)



Source: Author's own elaboration.

Temporal disaggregation and linear ordering

The article examines sparse temporal disaggregation and adaptive sparse temporal disaggregation methods by Mosley et al. [2022], both of which operate in high-dimensional settings using the least absolute shrinkage and selection operator (LASSO) of Tibshirani [1996] to exclude insignificant indicators from the model. Additionally, the study explores several classical temporal disaggregation techniques, including those by Denton [1971] and Chow and Lin [1971], and their extensions by Fernández [1981], Litterman [1983], Santos Silva and Cardoso [2001], alongside other variants detailed by Sax and Steiner [2013]. These methods are implemented under various assumptions regarding the specification of the distribution matrix and approaches to estimating the autoregressive parameter (Appendix: Table 1A).

The research also incorporates multiple univariate time series imputation methods as outlined by **Moritz and Bartz-Beielstein [2017]**. While temporal distribution methods rely on inter-attribute correlations, univariate time series imputation utilises time dependencies. This provides a benchmark to evaluate the performance of more complex temporal disaggregation techniques (Appendix: Table 2A).

Regression-based temporal disaggregation techniques utilise high-frequency auxiliary indicators to estimate missing high-frequency data points, aiming to ensure high precision and continuous historical data for the original low-frequency series. The natural indicator closely related to effective working time is average employment, calculated in full-time equivalents. In fact, average employment and time actually worked in a specific company for a given quarter are used as cross-checks by Statistics Poland. This study formally examines the relationship between hours actually worked and average employment using linear Pearson correlation, significance tests, bootstrap confidence intervals, descriptive statistics, and the Johansen cointegration test. Additionally, a range of unit root tests are applied, including ADF, KPSS, ADF-GLS, and PP tests. Most of the methods analysed, including the Chow-Lin method [**Fukuda, 2009**], perform particularly well when the auxiliary time series are closely associated with low-frequency series.

The study also investigates other potential quarterly auxiliary indicators: real Gross Domestic Product (GDP), real GDP *per capita*, the real Industrial Production Index (IPI), and real Gross Value Added (GVA). These indicators and their seasonal components, obtained using X-13ARIMA-SEATS, are examined, along with the consideration of lags specific to each economic activity section. To address the issue of incorporating lags and avoid delays in quarterly time worked estimates, missing data points are forecast using an updated version of the **Hyndman and Khandakar** [**2008**] algorithm. Additionally, a naive forecasting method is employed as a simple robustness check.

The objective of the temporal distribution in this study is to estimate an unknown high-frequency series of hours actually worked, y (in this case, quarterly), where the sums are consistent with a known low-frequency series, y_l, derived from an annual comprehensive enumeration survey (Form Z-06, as shown in Figure 1). In other contexts, the unknown high-frequency series may be aligned with the average, first or last values of a known low-frequency series. To estimate y, the methods can employ one or more high-frequency auxiliary variables (trackers) that are considered as proxies for y_l. The high-frequency auxiliary variables are collected in an $n \times m$ matrix X:

$$X = \{X_{j,t}: j = 1, ..., m, t = 1, ..., n\}, \tag{1}$$

where m represents the number of trackers, and n corresponds to the number of high-frequency observations (in this case, quarterly). In a specific case, the Denton and Denton-Cholette methods utilise a single indicator for the preliminary series, where $p=X$ and X is an $n\times 1$ matrix.

The temporal disaggregation methods involve generating a high-frequency preliminary time series p and subsequently allocating discrepancies between the annual preliminary and actual annual values y_l across the preliminary quarterly time series. The final estimation of the quarterly series is derived by adding the preliminary quarterly series to the distributed annual residuals, as outlined in Equation 2. The variety of temporal distribution methods can be simplified by organising them within a framework of equations (2) and (3) [**Sax, Steiner, 2013**]:

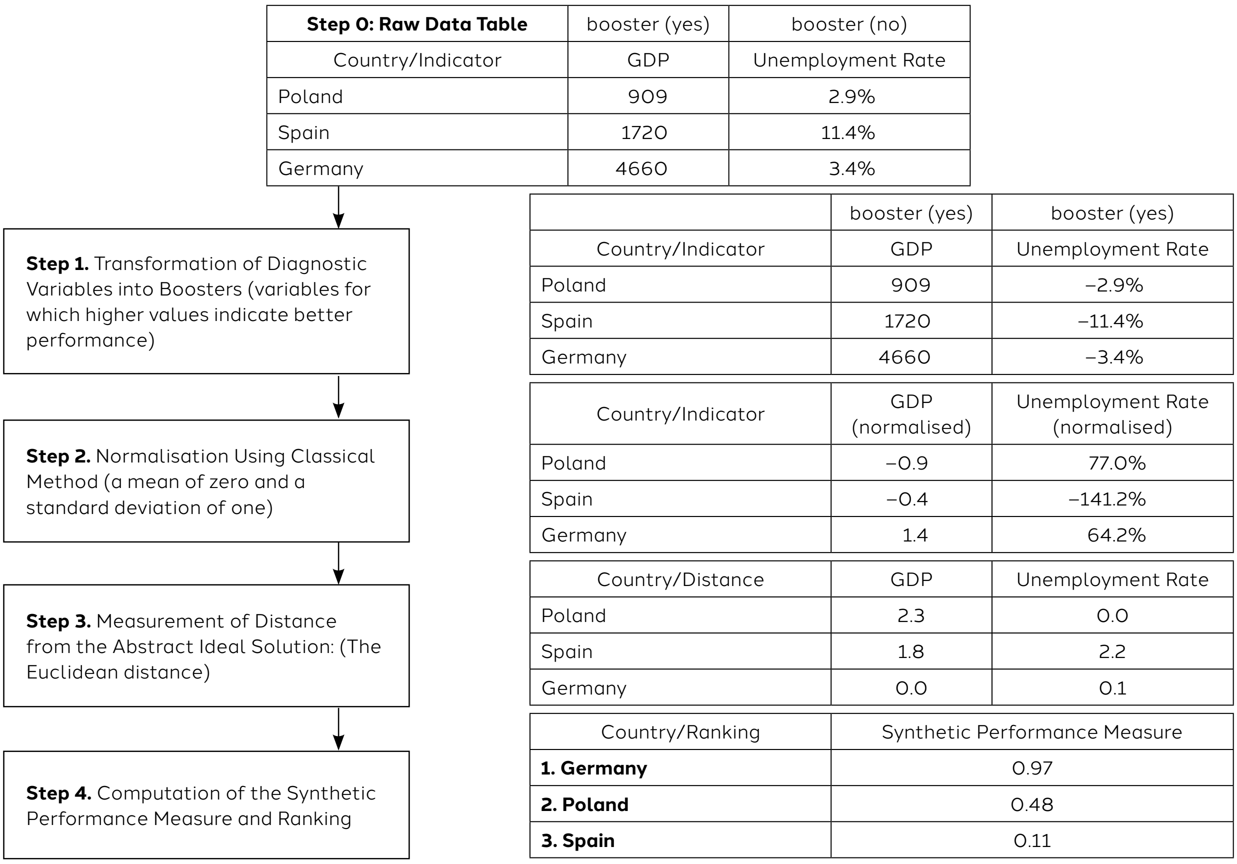
$$\hat{y} = p + Du_1 \qquad (2);$$

$$u_l \equiv y_l - Cp \qquad (3),$$

where: subscript l denotes low frequency variables, $\hat{y}$ is the final estimation of quarterly time series, D is the distribution matrix with dimensions equal to quarterly (n) and annual (n_l) observations, u_l is the vector of n_l length that consists of differences between the annualised values of p and y_l. C is the temporal aggregation – extrapolation matrix: $I_{n_l} \otimes \omega$, which is the Kronecker product of an identity matrix of size n_l and a transposed vector ω of length $\frac{n}{n_l}$ (frequency conversion ratio) which specifies the form of temporal aggregation. In this article, we set $\omega = [1, 1, 1, 1]$ meaning the annual values of hours actually worked represent the sum of the quarterly values. In other contexts, depending on the choice of ω, different cases may arise: annual values can represent the average of quarterly values if $\omega = [\frac{1}{n_l}, \frac{1}{n_l}, \ldots, \frac{1}{n_l}]$, annual values can equal the first quarterly value if $\omega = [1, 0, 0, \ldots, 0]$, or the last quarterly value if $\omega = [0, 0, \ldots, 1]$. Equation (2) serves as the foundation for the disaggregation methods, with the key differences among them lying in how they establish the preliminary series, p, and the distribution matrix, D [**Sax, Steiner, 2013**]. Please refer to the Appendix for a more detailed discussion of the methods developed by Chow and Lin, Fernández, and Litterman, as well as the dynamic extensions of the Chow-Lin approach.

The study evaluates forecast accuracy using Mean Error (ME), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Percentage Error (MPE), MAPE, and Theil's U [**Hyndman, Athanasopoulos, 2018**]. These are compared using the modified Diebold-Mariano test [**Diebold, Mariano, 2002**] corrected for small-sample bias [**Harvey et al., 1997**] and a taxonomic analysis based on the distance from the abstract ideal solution [**Hellwig, 1968**; **Walesiak, 2017**]. This approach is similar to the Technique for Order of Preference by Similarity to Ideal Solution [**Balcerzak, Pietrzak, 2017**], which identifies an abstract pattern. For alternative ordering methods, refer to the median-based version of the taxonomic analysis based on the distance from the abstract ideal solution by **Ryczkowski and Ręklewski** [**2021**], or refer to **Szczepaniak and Szulc-Obłoza** [**2020**], where the synthetic measure is obtained by computing the average of the final diagnostic variables. The list of additional methods for multivariate comparative analysis is not exhaustive and could be expanded to include techniques such as cluster analysis [**Lach, Malaga, 2023**].

Figure 2. Linear ordering method



Step 0: Raw Data Table	booster (yes)	booster (no)
Country/Indicator	GDP	Unemployment Rate
Poland	909	2.9%
Spain	1720	11.4%
Germany	4660	3.4%

Step 1. Transformation of Diagnostic Variables into Boosters (variables for which higher values indicate better performance)

	booster (yes)	booster (yes)
Country/Indicator	GDP	Unemployment Rate
Poland	909	−2.9%
Spain	1720	−11.4%
Germany	4660	−3.4%

Step 2. Normalisation Using Classical Method (a mean of zero and a standard deviation of one)

Country/Indicator	GDP (normalised)	Unemployment Rate (normalised)
Poland	−0.9	77.0%
Spain	−0.4	−141.2%
Germany	1.4	64.2%

Step 3. Measurement of Distance from the Abstract Ideal Solution: (The Euclidean distance)

Country/Distance	GDP	Unemployment Rate
Poland	2.3	0.0
Spain	1.8	2.2
Germany	0.0	0.1

Step 4. Computation of the Synthetic Performance Measure and Ranking

Country/Ranking	Synthetic Performance Measure
1. Germany	0.97
2. Poland	0.48
3. Spain	0.11

Notes: GDP for 2024, expressed in current billions of US dollars, was obtained from the International Monetary Fund (IMF), while the 2024 Labour Force Survey (LFS) unemployment rate was sourced from Eurostat. The ranking assumes equal weighting of GDP and the unemployment rate in the assessment.

Source: Author's own elaboration based on **Hellwig** [**1981**: 48] and **Walesiak** [**2016**].

Our linear ordering method (Figure 2) involves transforming diagnostic variables x_{kj} into boosters and normalising them using classical methods: $z_{kj} = \frac{x_{kj} - \bar{x}_j}{s_j(x)}$, where $\bar{x}_j$ is the mean, and $s_j(x)$ is the standard deviation. The variable M_j is considered a booster (stimulant), as initially defined by **Hellwig** [**1968**] if, for any two of its observations (real numbers for metric data and categories for ordinal data) x_{ij}^S and x_{kj}^S, referring to the objects (in this case, methods for determining quarterly time worked) A_i and A_k, it holds that $x_{ij}^S \succ x_{kj}^S \Rightarrow A_i \succ A_k$, where $\succ$ denotes the dominance of method A_i over method A_k (see, **Hellwig** [**1981**: 48], **Walesiak** [**2016**]). The diagnostic variables in our basic scenario include MPE to capture bias in the forecasts, MAPE to measure the average magnitude of error, and Theil's U to compare the accuracy of a forecast model with that of a naive model based on RMSE. In the primary scenario, the weights for the diagnostic variables were set equally.

The measure $m_i \in [0, 1]$ describes the closeness of the interpolation method to an abstract ideal solution. The further the value is from one, the more divergent the interpolated series is from the survey-based reference series. For $i \in (B, L)/\{I\}$ economic activity sections, the Theil's U and MPE diagnostic variables (where values further from $x_{Nj} = 0$, indicate worse performance) were transformed into boosters (where larger values indicate better performance) as follows:

$$\frac{-1}{x_{kj} - x_{Nj} - 1} \quad for\ x_{kj} < x_{Nj} = 0; \tag{11}$$

$$1 \quad for\ x_{kj} = x_{Nj} = 0; \tag{12}$$

$$\frac{1}{x_{kj} - x_{Nj} + 1} \quad for\ x_{kj} \geq x_{Nj} = 0, \tag{13}$$

where k and j denote the indices of interpolation methods and diagnostic variables used for evaluation respectively. The detractor MAPE (where larger values indicate worse performance) was transformed into booster by multiplying x_{kj} by –1.

The Euclidean distance d_{I0} measures how close the particular z_{ij} variable is to the abstract ideal solution z_j^P: $d_{I0} = \sqrt{\sum_{j=1}^{d} \left(z_{kj} - z_j^P\right)^2}$ (7). The final formula for m_i is as follows: $m_i = 1 - \frac{d_{I0}}{d_0}$ (8), where: d_0 is the total distance between the abstract ideal solution and the minimum z_j values for all of the disaggregation methods. The abstract ideal and inferior solutions are defined as the best and worst values of diagnostic variables achieved by the methods under consideration for a given economic activity section.

Data and preliminary statistical analysis

The quarterly time series of hours actually worked in medium-sized and large enterprises (>9 employees) are disseminated by Statistics Poland per employee for NACE Rev. 2 economic activity sections from B to L, excluding I, covering the period from the first quarter of 2009. Earlier data on time actually worked are either unpublished or not adjusted in accordance with the Polish Classification of Activities PKD-2007.

The total time actually worked (effective working time) is calculated by multiplying the officially published hours actually worked per employee by the average paid employment. Annual data on hours actually worked are available in the Statistical Yearbooks of Statistics Poland. Official data exclude entities with fewer than nine employees, individual agriculture, individuals employed abroad, and those engaged in social, political, and trade union organisations, as well as personnel involved in national defence and public security activities.

Effective working time encompasses both regular hours worked and overtime (excluding, for example, vacation and sick leave) by employees considered in the calculation of the average number of employed persons. Regular working hours are defined as the time worked during a standard workday, within the hours stipulated for a particular group of employees. This includes time spent on business trips, courses, and training sessions organised by the employer during working hours, whether conducted on-site or off-site. Additionally, regular working hours include time spent performing substitute tasks in the event of work stoppage at the employee's usual workstation. Periods during which an employee remains at the employer's disposal, meaning they can be called upon to work at any moment, are also considered paid working time, including during pandemics or periods of epidemic risk (for further details, see **Statistics Poland** [**2023**]).

Quarterly data on average employment are available through the basic macroeconomic indicators section on the website of Statistics Poland. Average paid employment includes individuals employed on a labour contract, converted to full-time equivalents. IPI is sourced from the OECD, while nominal GDP, GDP *per capita*, and GVA are obtained from Eurostat in national currency. As a deflator, we use the price index (2015 = 100) from Eurostat.

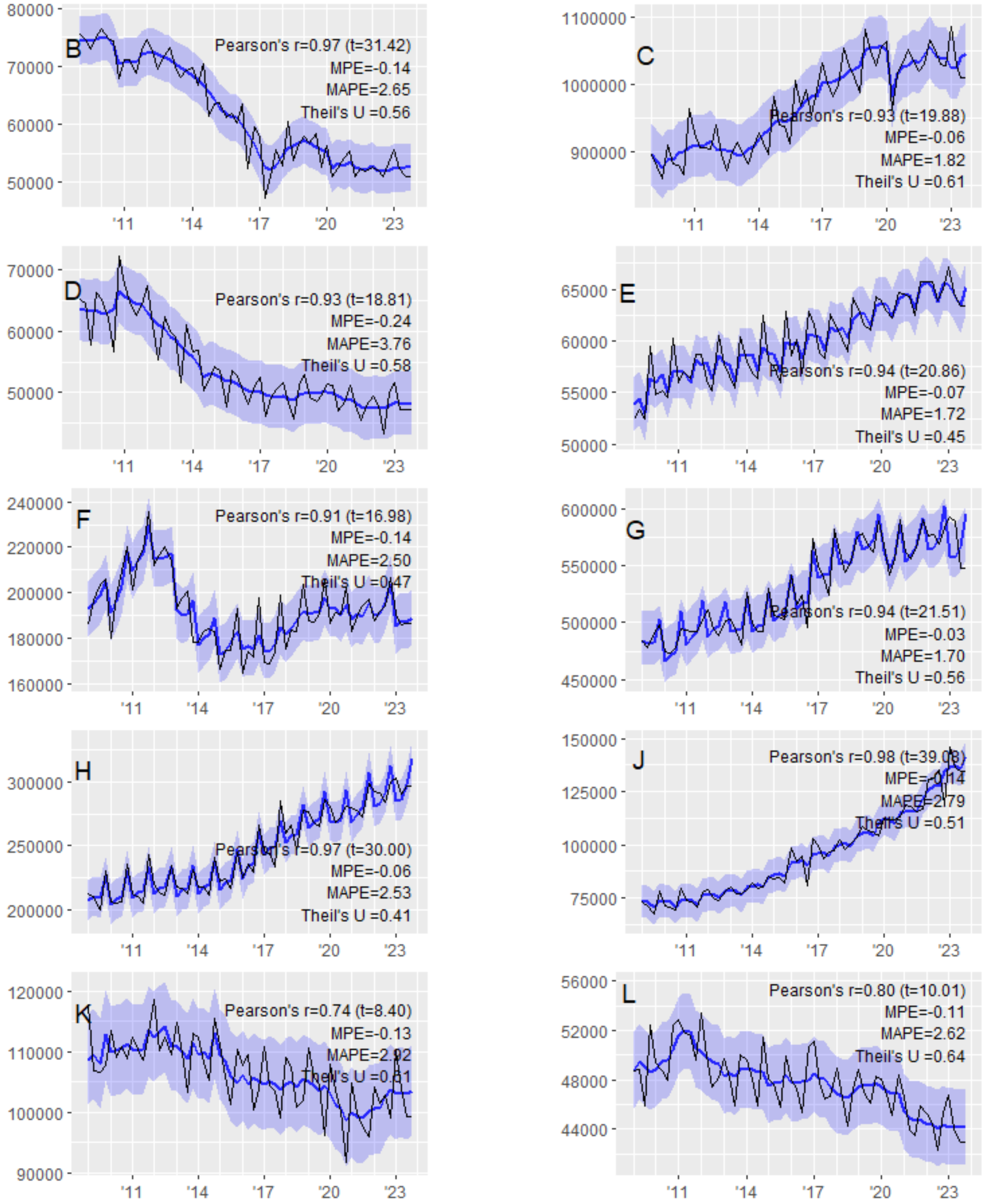
Preliminary data analysis, including the observation of common trends, correlations, unit roots, and linear cointegration, indicates a strong empirical relationship between average paid employment and time actually worked. The Pearson Correlation Coefficient between hours actually worked and average paid employment is statistically significant, ranging from 0.71 (for K economic activity section) to over 0.9 (for B, C, D, G, H, and J), with intermediate values of 0.76 (for L) and 0.87 (for E and F). The lower bound of the 0.95 bootstrap confidence interval for the Pearson Correlation Coefficient exceeds 0.8 on average.

Both time actually worked and average paid employment exhibit non-stationarity and are generally integrated of the same order, typically $I(2)$, as indicated by a comprehensive battery of unit root tests, including KPSS, ADF, ADF-GLS, and PP. The exception is found in section K, where six out of nine tests indicated $I(0)$ for time actually worked. Moreover, the null hypothesis of no cointegration between hours worked and average paid employment is rejected for all sections, except section G at the 0.05 significance level (and section K, due to the stationarity of hours worked); Appendix: Tables 3A-4A. Various versions of the Johansen cointegration tests, including those with unrestricted or restricted constants and trends, as well as sample size corrections for the Trace test, did not alter these results.

Empirical results

The **Litterman** [**1983**] method of temporal disaggregation, referred to as the Litterman-minrss [**Sax, Steiner, 2013**], which minimises the weighted residual sum of squares [**Barbone et al., 1981**], demonstrated the highest accuracy among the methods investigated. It achieved a MAPE ranging from approximately 1.60 to 1.69 percent in the E and G economic activity sections, respectively, to 3.76 percent in the D section – the only section in which the MAPE exceeded 3.0 percent. This method yielded these favourable outcomes when a regression on the low-frequency series was estimated using the average paid employment as the primary auxiliary indicator, supplemented by: (i) real GVA for sections H and F; (ii) real GDP for section G and its one-quarter lag for sections J and D (in the latter case, in *per capita* terms); (iii) the real industrial production index for section C; (iv) the seasonal components of real GDP *per capita* (derived via X-13ARIMA-SEATS), lagged by one quarter, for sections B and L; and (v) the seasonal component of real GVA, also lagged by one quarter, for sections E and K.

Figure 3. Litterman Temporal Distribution of Annual Time Actually Worked (in thousands of hours) to Quarterly Frequency across Economic Activity Sections in Poland from Q1 2009 to Q4 2023



Notes: The disaggregated (quarterly) high-frequency series on time actually worked are represented by blue lines, each surrounded by 95% confidence intervals. The black lines depict the actual Statistics Poland survey-based quarterly effective working time. The figure illustrates the Litterman method [1983] for temporal disaggregation, which involves minimising the weighted residual sum of squares [Barbone et al., 1981]. The temporal disaggregation is conducted using average employment as a high-frequency (quarterly) indicator series. For improved forecasts additional help indicators were used across economic activity sections: B, F- real Gross Value Added; L-real Gross Value Added lagged by one quarter; D,J,K- real GDP lagged by one quarter; H- difference between real GDP and its X-13ARIMA-SEATS seasonally adjusted series; E-difference between real GDP and its X-13ARIMA-SEATS seasonally adjusted series lagged by one quarter; G-difference between real Gross Value Added and its X-13ARIMA-SEATS seasonally adjusted series, C- real Industrial Production Index. The disaggregation assumes that the last value of the time actually worked (in the 4th quarter) aligns with the low-frequency (annual) series.

Source: Author's own elaboration.

The **Litterman** [**1983**] method also achieved favourable forecast accuracy when the average paid employment indicator was used as the *sole* auxiliary time series, with an average MAPE of 2.80% and our synthetic aggregate measure based on the distance from an abstract ideal solution of 0.979 [**Hellwig, 1968**; **Walesiak, 2017**]; Appendix: Figure 1A. Its maximum MAPE occurred in economic activity section D at 3.85%, and it exceeded 3.0% in two other sections, H and J. While the inclusion of additional auxiliary macroeconomic series alongside average paid employment improved forecast accuracy slightly (see Figure 3 and Table 6A for a comparison of MAPE across economic activity sections), relying solely on average paid employment ensures methodological simplicity, transparency, and consistency – features particularly valued in the production of official statistics (see Table 1).

Furthermore, the integration of supplementary macroeconomic auxiliary series presents practical challenges for official statistics. These indicators are typically published with significant delays, necessitating preliminary forecasts to enable their timely incorporation into official estimates. This issue becomes even more complex when employing lagged versions of such indicators, as forecasting errors can propagate into the disaggregation process. To address this issue, Table 6A employs two forecasting strategies – naive and ARIMA-based – with methodological details provided in the corresponding table note. However, doing so increases the methodological complexity, compromises transparency, and requires tailoring different auxiliary series to specific economic activity sections. An inappropriate selection of such indicators may result in considerable estimation errors (see Table 6A). By contrast, average paid employment data derived from administrative sources are both timely, readily available for use in official statistics, and closely related to time actually worked (see Section 5).

Given these considerations, this study recommends the use of average paid employment as the sole auxiliary series. This approach achieves only marginally lower accuracy compared to more complex alternatives, while significantly enhancing methodological coherence and applicability in official statistical contexts.

The empirical evidence in Table 1 shows that classical temporal disaggregation methods, along with their modern adaptations to high-dimensional settings, achieved high and comparable accuracy, denoted as grade A. Grade A was awarded when the synthetic aggregate measure m_i, based on MAPE, MPE, and Theil's U across all economic activity sections, exceeded the sum of its average and one standard deviation for all the methods investigated. Generally, the more complex methods, which involve greater computational effort and utilise the average paid employment indicator as an auxiliary time series, significantly outperformed the naive forecast, namely the Last Observation Carried Forward (Locf) method (grade: C, m_i=0.70).

Notably, the original **Denton** [**1971**] method performed only marginally better than the naive forecast (Locf). The poor performance of the Denton method stems from an unsatisfactory choice of initial conditions, which create transient movements at the beginning of the benchmarked series that violate the principle of movement preservation. The correction to the Denton method involves removing Denton's initial condition, which states that the first observation of the benchmarked series must equal the first element of the preliminary series [**Dagum, Cholette, 2006**]. Consequently, the denton-cholette method is preferable to the original denton method in most cases and, as shown in Table 1, is graded A with a MAPE of 2.84 percent.

The methods graded A in Table 1 outperformed not only simple computational methods (such as Locf or Stineman interpolation) but also more advanced techniques based on spline functions or the Kalman filter. Methods primarily concerned with movement preservation, including moving and simple averages, performed worse compared to temporal disaggregation methods, receiving grades B and C. Perhaps unexpectedly, simple linear interpolation based on the standard linear function demonstrated relatively satisfactory performance among movement preservation-focused methods (*grade*: *B*; $m_i = 0.83$).

Fast and simple imputation techniques, such as using the mode, median, or mean to replace missing data, yielded the least satisfactory results (grade D). These methods should generally be avoided for interpolation in public statistics and used with great caution in other contexts.

Table 1. Forecast accuracy of temporal distribution methods of annual time actually worked to quarterly frequency averaged across economic activity sections in Poland from Q1 2009 to Q4 2023

Method	$\bar{ME}$	$\bar{RMSE}$	$\bar{MAE}$	$\bar{MPE}$	$\bar{MAPE}$	max MAPE	$\bar{Theil's\ U}$	m_i	Grade
Litterman-minrss	−4.8E-09	7043.71	5491.70	−0.14	2.80	3.85	0.60	0.979	A
Litterman-fixed	−3.4E-11	7071.90	5517.42	−0.14	2.80	3.83	0.60	0.977	A
Chow-Lin- Quilis	4.23E-11	7085.05	5525.56	−0.14	2.81	3.82	0.61	0.976	A
Litterman-maxlog	−2E-12	7078.03	5520.30	−0.14	2.80	3.82	0.61	0.975	A
Fernandez	−2.4E-11	7083.27	5524.20	−0.14	2.81	3.82	0.61	0.975	A
Fast	2.58E-07	7083.38	5524.24	−0.14	2.81	3.82	0.61	0.975	A
Dynamic-maxlog	3.64E-12	7149.00	5607.45	−0.14	2.85	3.80	0.61	0.970	A
Chow-Lin- ecotrim	3.32E-12	7153.26	5564.38	−0.14	2.83	3.81	0.62	0.969	A
Chow-Lin-maxlog	1.3E-12	7150.54	5579.20	−0.14	2.84	3.80	0.62	0.969	A
Chow-Lin-fixed	4.49E-12	7160.27	5573.19	−0.14	2.84	3.81	0.62	0.968	A
Ols	3.41E-12	7159.91	5590.68	−0.14	2.85	3.80	0.62	0.968	A
Sparse	4.01E-12	7148.41	5552.60	−0.14	2.84	3.80	0.62	0.967	A
Adaptive	4.01E-12	7148.41	5552.60	−0.14	2.84	3.80	0.62	0.967	A
Denton-Cholette	8.28E-12	7157.65	5546.65	−0.14	2.84	3.81	0.62	0.966	A
Dynamic-minrss	– 3.2E-11	7411.65	5859.67	−0.14	2.92	3.83	0.62	0.957	A
Dynamic-fixed	3.37E-12	7424.95	5858.98	−0.15	2.93	3.95	0.62	0.954	A
Uniform	5.66E-12	7695.32	5970.50	−0.16	3.02	4.24	0.65	0.924	A
Nocb	3.06E-12	7697.39	5971.88	−0.16	3.03	4.24	0.65	0.924	A
Spline	214.90	7922.60	6192.74	−0.09	3.08	4.00	0.66	0.911	B
Stineman	811.23	7898.55	6065.55	−0.02	3.04	4.05	0.67	0.826	B
Linear	820.03	7901.22	6078.27	−0.02	3.05	4.06	0.67	0.825	B
EwMA	816.82	7950.97	6124.51	−0.02	3.06	4.04	0.67	0.824	B
Kalman 4	821.92	7899.34	6079.67	−0.01	3.05	4.06	0.67	0.824	B
LwMA	803.94	7842.61	6014.68	−0.03	3.05	4.10	0.67	0.823	B
SMA	792.72	7766.28	5917.29	−0.04	3.06	4.24	0.67	0.818	B
Kalman 1	1003.12	8150.27	6282.47	−0.02	3.10	4.05	0.68	0.793	C
Kalman 3	1003.12	8150.27	6282.47	−0.02	3.10	4.05	0.68	0.793	C
Kalman 2	1155.38	8766.94	6729.83	0.03	3.33	4.32	0.73	0.733	C
Denton	5.11E-10	16825.46	8958.83	−0.14	4.31	5.25	0.99	0.718	C
Locf	1640.06	8630.67	6645.80	0.12	3.41	4.60	0.74	0.703	C
Harmonic mean	1223.71	18195.45	15504.16	−0.19	7.52	15.40	1.42	0.561	D
Geometric mean	620.27	18201.12	15555.75	−0.58	7.60	15.73	1.44	0.489	D
Mean	5.82E-12	18241.91	15630.45	−0.99	7.71	16.18	1.46	0.442	D
Median	1266.142	18352.16	15532.41	0.47	7.46	15.53	1.43	0.434	D
Random	−1821.14	23159.22	18546.28	−2.76	9.70	23.63	1.97	0.219	D
Mode	17950.58	26395.95	19570.64	7.82	8.83	16.58	1.90	0.047	D

Notes: The temporal disaggregation is conducted using average employment as a sole high-frequency (quarterly) indicator series. The temporal distribution methods are ordered in descending order based on the synthetic aggregate measure m_i built across all the NACE Rev. 2 economic activity sections considered. If m_i equals 1, it indicates that, according to MPE, MAPE, and Theil's U, the interpolation method produced a time series that most closely matches the high-frequency time series obtained from statistical surveys. The metric "max MAPE" represents the highest MAPE produced by the method across all economic activity sections. The forecast errors with a dash above their names represent the respective average errors across all economic activity sections. The information on the methods used and their abbreviations are explained in Tables 1A-2A. The methods are graded using the positional grouping method (depending on the arithmetic average, $\bar{m}$, and standard deviation, s): Grade A: $\{m_i \in m\colon m_i > \bar{m} + s(m)\}$; Grade B: $\{m_i \in m\colon \bar{m} < m_{ii} \leq \bar{m} + s(m)\}$; Grade C: $\{m_i \in m\colon \bar{m} - s(m)(m) < m_i \leq \bar{m}\}$; Grade D: $\{m_i \in m\colon m_i \leq \bar{m} - s(m)\}$; Grade A consists of disaggregation methods with the best properties, while Grade D consists of disaggregation methods with the least favourable characteristics.

Source: Author's own elaboration.

However, what may initially seem surprising is the satisfactory performance of the Next Observation Carried Backward (Nocb) method, where missing values are interpolated using the values from the next available non-missing case that immediately follows the missing values (namely, inverted naive forecast); grade: A; $m_i = 0.924$. This indicates that in our research, data from the fourth quarters on time actually worked can be copied to the preceding three quarters of the same year, leading to relatively satisfactory forecast accuracy. This result is attributable to the characteristics of the data rather than any extraordinary properties of the Nocb method. It is likely that in other applications, Nocb would not perform as efficiently as in Table 1.

Nevertheless, the relatively satisfactory performance of Nocb confirms that naive forecasts can sometimes be surprisingly difficult (and occasionally impossible) to outperform [Gilliland, 2010]. We employ the Diebold-Mariano test for predictive accuracy (see Fiszeder et al. [2019]) to formally assess which interpolation methods surpass the inverted naive benchmark (Nocb) across economic activity sections, with the alternative hypothesis positing that the method investigated is more accurate than Nocb (Table 2).

For our preferred method (referred to as Litterman-minrss), even when utilising the average paid employment indicator as the sole auxiliary time series, the null hypothesis of equal forecast accuracy was rejected for six of 10 economic activity sections at the 0.05 significance level. Moreover, the method also outperformed Nocb in section C at the 0.06 significance level and was close to conventional significance levels in section H (p-value < 0.12). In sections J and K, the Litterman [1983] method and our naive Nocb benchmark demonstrated the same forecast accuracy.[1]

Table 2. Diebold-Mariano test for predictive accuracy from Q1 2009 to Q4 2023

Method	Nace Rev. 2 economic activity section									
	B	C	D	E	F	G	H	J	K	L
Litterman-minrss	**3.03 (0.00)**	1.55 (0.06)	**2.34 (0.01)**	**2.18 (0.02)**	**3.58 (0.00)**	**2.33 (0.01)**	1.18 (0.12)	−1.53 (0.93)	0.69 (0.25)	**2.01 (0.02)**
Litterman-fixed	**3.05 (0.00)**	1.52 (0.07)	**2.36 (0.01)**	**1.92 (0.03)**	**3.64 (0.00)**	**2.40 (0.01)**	0.57 (0.29)	−0.81 (0.79)	0.66 (0.26)	**2.00 (0.03)**
Chow-Lin-Quilis	**3.08 (0.00)**	1.55 (0.06)	**2.36 (0.01)**	**1.89 (0.03)**	**3.66 (0.00)**	**2.44 (0.01)**	0.54 (0.30)	−0.80 (0.79)	0.67 (0.26)	**2.03 (0.02)**
Litterman-maxlog	**3.08 (0.00)**	1.56 (0.06)	**2.36 (0.01)**	**1.92 (0.03)**	**3.61 (0.00)**	**2.44 (0.01)**	0.54 (0.29)	−0.80 (0.79)	0.67 (0.25)	**2.03 (0.02)**
Fernandez	**3.08 (0.00)**	1.56 (0.06)	**2.36 (0.01)**	**1.92 (0.03)**	**3.66 (0.00)**	**2.44 (0.01)**	0.54 (0.29)	−0.80 (0.79)	0.67 (0.25)	**2.03 (0.02)**
Fast	**3.08 (0.00)**	1.56 (0.06)	**2.36 (0.01)**	**1.92 (0.03)**	**3.66 (0.00)**	**2.44 (0.01)**	0.54 (0.30)	−0.80 (0.79)	0.66 (0.26)	**2.03 (0.02)**
Dynamic-maxlog	**2.65 (0.01)**	1.54 (0.06)	**2.38 (0.01)**	**1.86 (0.03)**	**3.77 (0.00)**	**2.50 (0.01)**	−0.14 (0.56)	−1.41 (0.92)	0.47 (0.32)	**1.70 (0.05)**
Chow-Lin-ecotrim	**3.09 (0.00)**	1.49 (0.07)	**2.36 (0.01)**	**1.90 (0.03)**	**3.68 (0.00)**	**2.34 (0.01)**	0.09 (0.47)	−0.88 (0.80)	0.29 (0.39)	1.65 (0.05)
Chow-Lin-maxlog	**2.65 (0.01)**	1.53 (0.07)	**2.38 (0.01)**	**2.03 (0.02)**	**3.68 (0.00)**	**2.33 (0.01)**	0.11 (0.46)	−0.76 (0.77)	0.29 (0.39)	**1.94 (0.03)**
Chow-Lin-fixed	**3.09 (0.00)**	1.49 (0.07)	**2.36 (0.01)**	**1.90 (0.03)**	**3.70 (0.00)**	**2.34 (0.01)**	0.03 (0.49)	−0.86 (0.80)	0.28 (0.39)	1.59 (0.06)
Ols	**2.65 (0.01)**	1.54 (0.06)	**2.38 (0.01)**	**2.03 (0.02)**	**3.77 (0.00)**	**2.50 (0.01)**	−0.14 (0.56)	−0.76 (0.77)	0.37 (0.36)	**1.70 (0.05)**
Sparse	**2.95 (0.00)**	1.52 (0.07)	**2.37 (0.01)**	**1.68 (0.05)**	**3.64 (0.00)**	**2.39 (0.01)**	−0.23 (0.59)	−0.87 (0.81)	−0.18 (0.57)	1.68 (0.05)
Adaptive	**2.95 (0.00)**	1.52 (0.07)	**2.37 (0.01)**	**1.68 (0.05)**	**3.64 (0.00)**	**2.39 (0.01)**	−0.22 (0.59)	−0.87 (0.81)	−0.18 (0.57)	1.63 (0.05)

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[1] In J: Litterman method (ME=–4.72E-08; RMSE=4343.2; MAE=3214.5; MPE=–0.18; MAPE=3.4; Theil's U=0.60); Nocb (ME=4.85E-13; RMSE=3910.3; MAE=2782.0; MPE=–0.18; MAPE=3.0; Theil's U=0.58). In K: Litterman method (ME=–1.51E-10; RMSE=3834.6; MAE=3145.4; MPE=–0.13; MAPE=3.0; Theil's U=0.62); Nocb (ME=–1.94E-12; RMSE=3940.3; MAE=3227.7; MPE=–0.14; MAPE=3.0; Theil's U=0.64).

cont. Table 2

Method	Nace Rev. 2 economic activity section									
	B	C	D	E	F	G	H	J	K	L
Denton-Cholette	**3.07** **(0.00)**	1.60 (0.06)	**2.35** **(0.01)**	**1.70** **(0.05)**	**3.67** **(0.00)**	**2.44** **(0.01)**	−0.29 (0.61)	−0.98 (0.83)	−0.30 (0.62)	1.64 (0.05)
Dynamic-minrss	**2.80** **(0.00)**	−0.04 (0.52)	**2.42** **(0.01)**	**2.23** **(0.01)**	**2.20** **(0.02)**	**1.92** **(0.03)**	**2.35** **(0.01)**	−1.54 (0.94)	0.88 (0.19)	1.51 (0.07)
Dynamic-fixed	**2.83** **(0.00)**	0.01 (0.50)	**2.44** **(0.01)**	**2.12** **(0.02)**	**3.08** **(0.00)**	**2.02** **(0.02)**	1.28 (0.10)	−1.32 (0.90)	0.48 (0.32)	1.07 (0.14)
Uniform	**3.11** **(0.00)**	**1.96** **(0.03)**	**2.63** **(0.01)**	**2.25** **(0.01)**	**3.79** **(0.00)**	**2.68** **(0.00)**	0.31 (0.38)	0.97 (0.17)	1.66 (0.05)	**2.33** **(0.01)**
Spline	0.93 (0.18)	−1.55 (0.94)	**1.78** **(0.04)**	−1.85 (0.97)	−0.70 (0.76)	−0.28 (0.61)	−0.17 (0.57)	−2.57 (0.99)	1.04 (0.15)	0.34 (0.37)
Stineman	−0.59 (0.72)	−1.09 (0.86)	0.19 (0.43)	1.44 (0.08)	−0.07 (0.53)	0.43 (0.33)	**1.70** **(0.05)**	−2.31 (0.99)	0.48 (0.32)	−0.38 (0.65)
Linear	−0.64 (0.74)	−1.12 (0.87)	0.16 (0.44)	1.57 (0.06)	0.03 (0.49)	0.36 (0.36)	**1.69** **(0.05)**	−2.30 (0.99)	0.34 (0.37)	−0.55 (0.71)
EwMA	−0.57 (0.71)	−1.25 (0.89)	0.43 (0.33)	0.97 (0.17)	−0.06 (0.52)	0.11 (0.49)	1.33 (0.09)	−2.36 (0.99)	0.45 (0.33)	−0.42 (0.66)
Kalman 4	−0.64 (0.74)	−1.12 (0.87)	0.19 (0.42)	1.57 (0.06)	0.05 (0.48)	0.37 (0.36)	**1.81** **(0.04)**	−2.33 (0.99)	0.34 (0.37)	−0.55 (0.71)
LwMA	−0.80 (0.79)	−0.97 (0.83)	−0.29 (0.61)	0.93 (0.18)	0.06 (0.48)	0.79 (0.22)	**2.20** **(0.02)**	−2.18 (0.98)	0.16 (0.44)	−0.94 (0.82)
SMA	−1.19 (0.88)	−0.68 (0.75)	−1.03 (0.85)	0.78 (0.22)	−0.02 (0.51)	1.27 (0.11)	**2.82** **(0.00)**	−1.80 (0.96)	−0.38 (0.65)	−1.66 (0.95)
Kalman 1	−0.76 (0.77)	−2.27 (0.99)	0.22 (0.41)	−2.48 (0.99)	0.048 (0.48)	0.12 (0.45)	**1.81** **(0.04)**	−2.26 (0.99)	1.14 (0.13)	−0.12 (0.55)
Kalman 3	−0.76 (0.77)	−2.27 (0.99)	0.22 (0.41)	−2.48 (0.99)	0.05 (0.48)	0.12 (0.45)	**1.81** **(0.04)**	−2.26 (0.99)	1.14 (0.13)	−0.12 (0.55)
Kalman 2	−2.12 (0.98)	−2.21 (0.98)	−0.46 (0.68)	−0.75 (0.77)	−1.92 (0.97)	−1.10 (0.86)	1.21 (0.12)	−2.36 (0.99)	−0.91 (0.82)	−2.22 (0.98)
Denton	−1.40 (0.92)	−1.52 (0.93)	−1.38 (0.91)	−1.53 (0.93)	−1.32 (0.90)	−1.46 (0.93)	−1.50 (0.93)	−1.84 (0.96)	−1.44 (0.92)	−1.45 (0.92)
Locf	−2.17 (0.98)	−1.77 (0.96)	−1.71 (0.95)	**1.79** **(0.04)**	−1.98 (0.97)	−0.38 (0.64)	1.19 (0.12)	−2.83 (0.99)	−1.12 (0.87)	−2.31 (0.99)
Harmonic mean	−6.56 (1.0)	−6.89 (1.0)	−4.08 (1.0)	−5.53 (1.0)	−3.96 (.999)	−6.94 (0.99)	−6.96 (1.0)	−4.75 (1.0)	−2.14 (0.98)	−3.06 (0.99)
Geometric mean	−6.85 (1.0)	−6.91 (1.0)	−4.24 (1.0)	−5.55 (1.0)	−4.02 (.99)	−6.97 (0.99)	−7.27 (1.0)	−5.08 (1.0)	−2.16 (0.98)	−3.07 (0.99)
Mean	−7.12 (1.0)	−6.91 (1.0)	−4.43 (1.0)	−5.56 (1.0)	−4.09 (.99)	−6.98 (0.99)	−7.51 (1.0)	−5.50 (1.0)	−2.11 (0.98)	−3.09 (0.99)
Median	−5.75 (1.0)	−6.82 (1.0)	−3.49 (1.0)	−5.45 (1.0)	−4.12 (.99)	−6.87 (0.99)	−6.07 (1.0)	−4.89 (1.0)	−2.4 (0.99)	−3.17 (0.99)
Random	−4.26 (1.0)	−5.15 (1.0)	−5.30 (1.0)	−4.71 (1.0)	−3.82 (.99)	−4.60 (0.99)	−5.71 (1.0)	−5.14 (1.0)	−2.80 (0.99)	−4.44 (0.99)
Mode	−5.28 (1.0)	−5.49 (1.0)	−3.91 (1.0)	−5.13 (1.0)	−3.70 (.99)	−5.31 (0.99)	−4.66 (1.0)	−4.48 (1.0)	−5.18 (1.0)	−4.52 (1.0)

Note: *p-values in brackets, against the Next Observation Carried Backward* The null hypothesis is that the method under investigation and the inverted naive benchmark (Next Observation Carried Backward) have the same forecast accuracy. The alternative hypothesis is that the method under investigation is more accurate than the inverted naive benchmark. Bolded values indicate that the null hypothesis is rejected at the 0.05 significance level. A positive value of the Diebold-Mariano statistic ($DM > 0$) – with higher values indicating greater relative performance – suggests that the naive benchmark yields higher average forecast errors than the evaluated model. The temporal distribution methods are ranked in descending order based on the synthetic aggregate measure m_i as shown in Table 1. The temporal distribution methods are with the average paid employment indicator as the sole auxiliary time series.

Source: Author's own elaboration.

An important question remains as to whether the COVID-19 pandemic or the global energy crisis may have affected the forecast accuracy of the preferred Litterman method. Figure 3 suggests that the **Litterman**

[1983] method remains largely resilient to the impacts of the COVID-19 pandemic and the subsequent global energy crisis. Formal forecast accuracy metrics generally support this observation (Table 3).

The average MAPE across the economic activity sections remained comparable between the period from Q1 2009 to Q4 2023 and the subsample from Q1 2020 (following the World Health Organisation's declaration of a public health emergency due to COVID-19 on January 30) to Q4 2023, showing a slight reduction from 2.49% to 2.36%. The average MPE also decreased from –0.13 to –0.08. However, in the subsample from Q1 2020 onwards, temporal distribution led to an increase in the RMSE, resulting in the average Theil's U rising from 0.54 to 0.65 (see Table 3 and Figure 3).

Table 3. Forecast accuracy across economic activity sections for Litterman temporal distribution of annual time actually worked (in thousands of hours) to quarterly frequency across economic activity sections in Poland from Q1 2020 to Q4 2023

Nace Rev. 2	ME	RMSE	MAE	MPE	MAPE	ACF1	Theil's U
B	–1.73E-10	1359.16	1114.39	–0.07	2.10	0.09	0.60
C	8.27E-09	22578.76	16489.11	–0.05	1.59	0.07	0.59
D	–6.68E-10	1930.58	1611.65	–0.18	3.36	–0.09	0.63
E	4.61E-10	719.36	487.25	–0.02	0.75	–0.15	0.46
F	–4.78E-10	3725.54	3111.80	–0.04	1.62	–0.09	0.40
G	4.39E-09	18783.48	13349.74	–0.06	2.34	0.16	0.99
H	–1.77E-09	10635.34	8908.46	0.00	3.07	–0.12	1.00
J	–2.92E-07	5756.85	4021.45	–0.13	3.17	–0.46	0.60
K	2.26E-10	3895.60	3211.22	–0.15	3.17	–0.16	0.60
L	–7.77E-10	1254.02	1088.84	–0.08	2.42	0.04	0.67
average	–2.83E-08	7063.87	5339.39	–0.08	2.36	–0.07	0.65
s	8.80872E-08	7393.30	5346.08	0.06	0.81	0.17	0.19

Notes: The table illustrates the forecast accuracy measures for the Litterman method [1983] of temporal disaggregation, which involves minimising the weighted residual sum of squares [Barbone et al., 1981]. The temporal disaggregation is conducted using average employment as a high-frequency (quarterly) indicator series and additional optimal help indicators identified across economic activity sections, as shown in Figure 3.

Source: Author's own elaboration.

As an additional robustness check, Table 7A in the Appendix presents the respective forecast accuracy metrics for the pre-COVID period (Q1 2009–Q4 2019). When comparing this period to the COVID-affected subsample (Q1 2020–Q4 2023), MAPE decreased in four economic activity sections (G, H, J, K), increased in five (B, C, D, E, F), and remained roughly unchanged in one (L). Overall, the average MAPE across all sections remained relatively stable, increasing only marginally by 0.15 percentage points in the pre-COVID period.

The resilience of the Litterman method to the impacts of the COVID-19 pandemic and the subsequent global energy crisis may be explained by the fact that persistent external shocks ultimately influence the annual data. This annual data is then "split" into quarterly data using temporal distribution methods, allowing the quarterly data to reflect the impact of the external shock. Furthermore, this robustness may partly arise from the strong empirical relationship between average paid employment and effective working time (as discussed in Section 5). If the effects of the crises were reflected in average paid employment – the key auxiliary time series used for temporal disaggregation – then the temporal distribution methods could effectively capture the corresponding changes in effective working time.

Limitations

If the method proposed in this article were adopted, it would eliminate the need for the Z-03 survey. However, the final data on hours actually worked for the first, second, and third quarters of a given year would only be available at the beginning of the following year, once the annual data from Form Z-06 have been processed to enable temporal disaggregation into quarterly data (as illustrated in the lower section of Figure 1). If earlier quarterly data on effective working time were required for official statistics, the method outlined in this article would need to be supplemented by estimating preliminary data for the first three quarters of the given year. In this case, our method would consist of first estimating preliminary data and then finalising them. This procedure should be conducted separately for each NACE Rev. 2 economic activity section.

Although determining a method for extrapolating quarterly hours actually worked is not the focus of this article, Table 1 suggests that a straightforward approach for estimating preliminary data on hours worked for the first, second, and third quarters of a given year – before the annual data from Form Z-06 becomes available – could be the Last Observation Carried Forward (LOCF) method (Table 2A). The LOCF approach demonstrates a relatively good performance, with an average MAPE of 3.4% across economic activity sections (Table 1).

An alternative and more sophisticated method may involve using ARIMA models for forecasting. For instance, Figure 2A illustrates ARIMA-based forecasts for the first three quarters of 2023 with an average MAPE of 3.4%. ARIMA models were selected based on the Akaike Information Criterion, with an estimate of seasonal strength as per **Wang et al.** [**2006**]. Model selection followed the algorithm presented in **Hyndman and Khandakar** [**2008**]. In most cases, the data on hours actually worked from Form Z-03 for Q1, Q2, and Q3 of 2023 fall within the 95% confidence interval (Appendix: Figure 2A). A similarly strong forecasting performance was observed over the preceding five years, including 2020, when the COVID-19 pandemic began. It is possible that alternative or more sophisticated methods could provide even more accurate forecasts, making this an excellent topic for future research, contingent on the need for preliminary data by official statistics.

The temporal distribution techniques have also other limitations. First, they do not account for variations in time actually worked due to employee holidays and sick leaves. These factors likely exhibit a seasonal pattern that is not adequately captured by macroeconomic auxiliary variables such as average paid employment. Additionally, the relationship between effective working time and average employment, while close, is not one-to-one. Various factors contribute to this discrepancy, including high employee turnover, rounding of conversion factors to full-time equivalents, and the method of calculating average employment without accounting for daily stocks.

These limitations could be mitigated by using administrative data sources. Although data on holiday leaves is not available from administrative registers, these registers could provide quarterly estimates of hours *not* worked due to sick leaves in a given economic activity section. The actual time worked in quarters, as obtained through temporal distribution methods, could then be adjusted proportionally to the estimated quarterly length of sick leaves. Finally, the administrative registers could help more accurately account for employee turnover based on daily stocks.

Another potential solution could involve introducing discretionary permanent adjustments based on historical dependencies to reflect historical seasonality. However, such adjustments do not account for structural changes in the number of hours actually worked in a given economic activity section. Variations in sick leaves, holiday leaves, and other potential changes in the seasonal pattern of effective working time can have multiple causes and cannot be ruled out. Furthermore, temporal distribution methods will not capture external shocks with durations shorter than one year unless these shocks are reflected in the time actually worked in the fourth quarters or in the auxiliary variables used in the analysis.

Despite these limitations, the forecast accuracy measures derived from the Litterman method were found to be acceptable according to standard criteria established in the literature. Consequently, this method, along with others demonstrating similarly high forecast accuracy, could serve as a viable alternative to costly and time-consuming quarterly surveys.

Conclusions

Unlike employment and wage statistics, data on effective working time within enterprises is typically not available from administrative sources and must be obtained through surveys or econometric methods. The latter approach would benefit the state budget, reduce the reporting burden on enterprises, and decrease energy consumption. As a result, enterprises could allocate their time more productively, rather than spending it on time-consuming quarterly reports. Sustainable statistical practices should seek to replace surveys with alternative methods or data sources. Despite their potential, temporal disaggregation methods remain underutilised for estimating statistics on hours actually worked in several NSOs.

This article assesses the effectiveness of various methods for replacing official Statistics Poland quarterly data on time actually worked within enterprises in Poland, covering the period from the first quarter of 2009 to the fourth quarter of 2023. The analysis focuses on enterprises with more than nine employees, including public sector units regardless of size, across NACE Rev. 2 economic activity sections. The methods investigated include recent high-dimensional approaches proposed by **Mosley et al.** [**2022**], univariate time series imputation techniques by **Moritz and Bartz-Beielstein** [**2017**], and classical temporal disaggregation methods [Sax and Steiner, 2013]. These methods were selected for their practicality, computational viability, resilience, ease of understanding, statistical integrity, and adaptability, which is important in official statistics.

The study innovatively applied principles of Polish taxonomic thought to rank interpolation methods using a taxonomic analysis based on the distance from an abstract ideal solution, as outlined by **Hellwig** [**1968**] and **Walesiak** [**2017**]. Unlike many empirical studies that focus on a single method or only a few, this study provides a comprehensive assessment of several methods using a broad range of forecast accuracy metrics, tests, auxiliary variables, their combinations, and lags.

The findings suggest prioritising temporal distribution methods with the average paid employment indicator as an auxiliary time series, rather than those focused solely on movement preservation. Classical temporal distribution methods and their modern adaptations achieved similar accuracy in estimating quarterly effective working time. However, the **Litterman** [**1983**] method, with the minimisation of the weighted residual sum of squares [**Barbone et al., 1981**], exhibited the best forecast accuracy. The method remained resilient to the impacts of the COVID-19 pandemic and the subsequent global energy crisis. Despite the recognised challenges in the literature of outperforming naive forecasts in some applications, the method significantly outperformed the Next Observation Carried Backward method in the majority of economic activity sections. Employing additional macroeconomic auxiliary indicators further improved the forecast (average MAPE decreased from 2.8% to 2.5%). However, the optimal auxiliary indicator varied across economic activity sections. Using solely the average paid employment indicator as an auxiliary time series provided satisfactory results while maintaining simplicity, ease of understanding, and consistency across economic activity sections.

In an era where timely and accurate economic data is crucial for policy makers, businesses, and researchers, the study's findings provide valuable insights for selecting optimal temporal disaggregation methods in labour market statistics. Future research could explore combining temporal distribution methods with administrative data on sick leaves to more accurately capture seasonal patterns in time actually worked.

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Appendix

Table 1A. Temporal distribution methods

Name / abbreviation	References and short description
Sparse	Sparse temporal disaggregation of **Mosley et al.'s** [**2022**] high-dimensional extension of the regression-based techniques proposed by **Chow and Lin** [**1971**].
Adaptive	Adaptive sparse temporal disaggregation of **Mosley et al.'s** [**2022**] high-dimensional extension of the regression-based techniques proposed by **Chow and Lin** [**1971**]. The adaptive LASSO is particularly useful if the columns of the design matrix reveal multicollinearity.
Chow-Lin-maxlog	**Chow and Lin** [**1971**] with the lower bound for the autoregressive parameter $\rho = 0$; autoregressive parameter estimated using GLS
Chow-Lin-ecotrim	**Chow and Lin** [**1971**] with the minimisation of the weighted residual sum of squares, using a correlation matrix instead of the variance covariance matrix [**Barcellan, 2003**]. Autoregressive parameter estimated using GLS estimated. Chow-Lin-minrss-ecotrim in **Sax and Steiner** [**2013**].
Chow-Lin-Quilis	The Matlab extension of **Quilis** [**2012**] which multiplies the correlation matrix with $1/(1 - \rho^2)$. An autoregressive parameter is GLS estimated. Chow-Lin-minrss-Quilis in **Sax and Steiner** [**2013**].
Chow-Lin-fixed	**Chow and Lin** [**1971**] for a predefined autoregressive parameter $\rho = 0.5$. The correlation matrix is used instead of the variance-covariance matrix.
Dynamic-maxlog	Methods based on dynamic extensions to the original **Chow and Lin** [**1971**] method proposed by **Silva and Cardoso** [**2001**].
Dynamic-minrss	
Dynamic-fixed	
Fernandez	**Fernández** [**1981**].
Litterman-maxlog	**Litterman** [**1983**] for the autoregressive parameter $\rho = 0$, an autoregressive parameter is GLS estimated.
Litterman-minrss	**Litterman** [**1983**] with the minimisation of the weighted residual sum of squares [**Barbone et al., 1981**], an autoregressive parameter is GLS estimated.
Litterman-fixed	**Litterman** [**1983**] for an invariant truncation parameter $\rho = 0.5$. The correlation matrix is used instead of the variance-covariance matrix.
Denton-Cholette	**Dagum and Cholette** [**2006**] eliminate the transient movement present in the original "denton" method at the start of the resulting series.
Denton	**Denton's** [**1971**] technique can minimise the sum of squares of the deviations between the levels, the first differences, or the second differences of the indicator and the derived series.
Fast	chow-lin-fixed with fixed.rho = 0.99999. It returns roughly the same results as denton-cholette, which minimises the sum of squares of the deviations between the first differences of indicator and the derived series.
Uniform	The "uniform" method stands for a special case of the "denton" approach, with the order of differencing equal to 0 and the minimisation criterion that distributes the residuals uniformly.
Ols	The "ols" function conducts an ordinary least squares (OLS) regression and distributes the residuals uniformly, which allows for comparisons of the estimators from both the GLS and OLS regressions.

Notes: The names of the methods are close to original following Sax and Steiner [2013] to preserve consistency and prevent the reader's confusion as to which exact method is used.

Source: Sax and Steiner [2013]; Mosley et al. [2022].

Table 2A. Imputation methods

Name / abbreviation	References and short description
Simple imputation techniques (simple but fast approaches)	
Spline	Uses spline interpolation to replace missing values. For a given interpolated interval, four intervals are created and in each of them the function is interpolated with an interpolation polynomial [**Forsythe et al., 1977**].
Stineman	Uses Stineman interpolation to replace missing values [**Stineman, 1980**].
Kalman 1; *Kalman 2;* *Kalman 3;* *Kalman 4.*	Interpolation with the Kalman filter using a simple structural model (so-called State-Space Model) estimated by the maximum likelihood method. In the article, we consider: Kalman 1: Kalman Smoothing is used for estimation; Kalman 2: Kalman Run is used for estimation; Kalman 3: Kalman Smoothing is used for estimation based on the structural model; Kalman 4: Kalman Smoothing is used for estimation based on the structural model with a trend.
Linear Weighted Moving Average (LwMA)	Replaces missing values with moving average values. The width of the moving average window is 4 quarters. Weighting successive observations directly to the central value reduces the arithmetic progression as follows: ½, 1/3, ¼, and so on.

cont. Table 2A

Name / abbreviation	References and short description
Exponential Weighted Moving Average (EwMA)	Replaces missing values with moving average values. The width of the moving average window is 4 quarters. Weighting factors directly to the central value decrease exponentially as follows: $1/2^1$; $1/2^2$; $1/2^3$; ..., and so on.
Simple Moving Average (SMA)	Replaces missing values with moving average values. The width of the moving average window is 4 quarters, with equal weights for all observations when calculating the mean.
Geometric mean	Replaces missing values with the geometric mean of all the non-missing values.
Harmonic mean	Replaces missing values with the harmonic mean of all the non-missing values.
More advanced algorithms that require more computation	
Linear	Standard approximation using a linear function.
Last Observation Carried Forward (Locf)	Interpolates missing values using the values from the last available non-missing observation. In other words, this is known as a naive forecast.
Next Observation Carried Backward (Nocb)	Involves a method similar to LOCF but operates in the opposite direction. Missing values are interpolated using the values from the next available non-missing case that immediately follows the missing values. In other words, this is referred to as an inverted naive forecast.
Mode	Replaces missing values with the most frequent value in the time series. If two or more values had the same frequency, the lower value was chosen.
Median	Replaces missing values with the median of all the non-missing values.
Mean	Replaces missing values with the arithmetic mean of all the non-missing values.
Random	Replaces each missing value by randomising a number within the boundaries defined by the minimum and maximum values of the non-missing data in a given time series. We expect this method to deliver inferior results.

Source: Moritz and Bartz-Beielstein [2017].

Table 3A. Johansen test for cointegration of the worked time and average paid employment in Poland, Q1 2009–Q4 2023

NACE Rev. 2	Trace test [*p*-value]	Eigenvalues	Lmax test [*p*-value]
B	20.341 [0.0247]	0.28818	19.036 [0.0238]
C	22.038 [0.0133]	0.28887	19.090 [0.0234]
D	34.934 [0.0001]	0.43902	32.372 [0.0001]
E	21.789 [0.0146]	0.28737	18.973 [0.0244]
F	29.719 [0.0006]	0.33977	23.249 [0.0046]
G	15.725 [0.1136]	0.19564	12.192 [0.2310]
H	43.301 [0.0000]	0.49560	38.326 [0.0000]
J	19.914 [0.0287]	0.28201	18.553 [0.0285]
L	29.621 [0.0007]	0.29614	19.665 [0.0188]

Notes: Test with unrestricted trend and constant. Seasonal dummies are included. Lag order = 4. The null hypothesis (H0) states that there are no cointegrating vectors.

Source: Author's own elaboration.

Table 4A. Unit root tests (alfa = 0.05) for the quarterly time worked and average paid employment in Poland for national economic sections from Q1 2009 to Q4 2023

NACE Rev. 2 Section	KPSS with const. (0.46)	KPSS with const. and linear trend (0.15)	ADF without const. (−1.95)	ADF with const. (−2.89)	ADF with const. and linear trend (−3.45)	ADF-GLS with const. (−1.95)	ADF-GLS with const. and linear trend (−3.03)	PP with const. (−2.91)	PP with const. and linear trend (−3.49)
Average paid employment									
B	1.5	0.25	−1.46	−1.39	−1.3	−0.2	−1.26	−1.33	−0.79
ΔB	0.25	0.12	−1.26	−1.6	−3.03	−1.34	−1.76	−6.78	−6.95
C	1.48	0.18	1.84	−0.83	−1.1	−0.49	−1.54	−0.37	−2.25
ΔC	0.19	0.18	−3.8	−4.23	−4.22	−0.5	−1.02	−6.62	−6.53

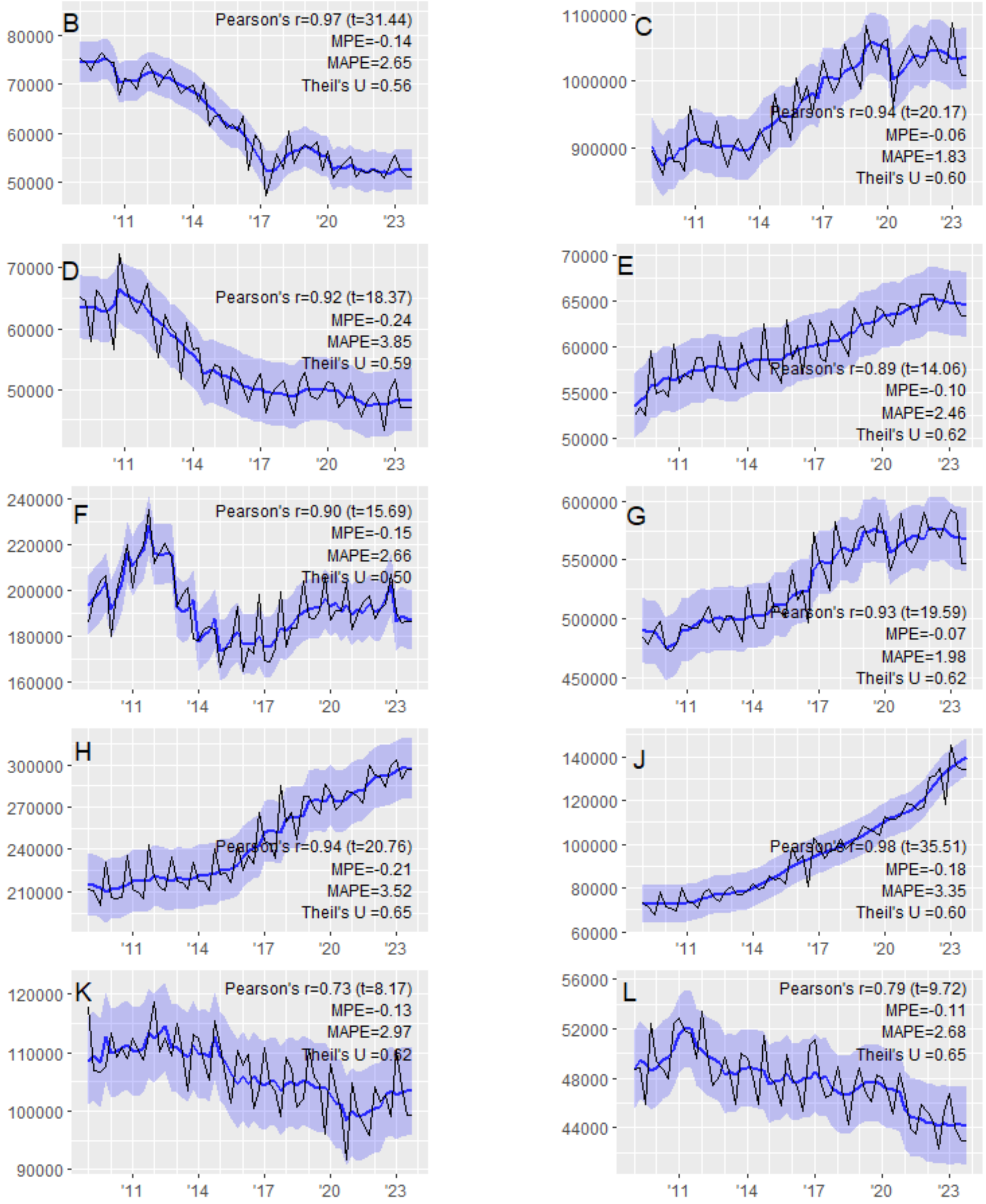
cont. Table 4A

NACE Rev. 2 Section	KPSS with const. (0.46)	KPSS with const. and linear trend (0.15)	ADF without const. (−1.95)	ADF with const. (−2.89)	ADF with const. and linear trend (−3.45)	ADF-GLS with const. (−1.95)	ADF-GLS with const. and linear trend (−3.03)	PP with const. (−2.91)	PP with const. and linear trend (−3.49)
D	1.4	0.3	−1.32	−4.56	−3.25	−0.56	−1.81	−1.24	−0.87
ΔD	0.24	0.12	−1.54	−1.66	−5.03	−1.52	−1.88	−5.87	−5.92
E	1.58	0.22	4.11	0.64	−3.37	1.18	−3.37	−1.12	−2.83
ΔE	0.12	0.1	−2.2	−6.2	−6.31	−0.2	−1.1	−11.39	−11.33
F	0.48	0.23	−1.06	−3.95	−3.85	−1.95	−2.34	−1.5	−1.65
ΔF	0.14	0.14	−2.74	−2.85	−3.05	−1.44	−1.95	−9.22	−9.13
G	1.53	0.17	2.38	−0.6	−2.24	0.02	−1.58	−0.34	−1.77
ΔG	0.15	0.14	−1.79	−4.28	−4.24	−1.33	−1.59	−6.93	−6.87
H	1.51	0.33	1.53	−0.31	−2.45	−0.37	−1.31	1.4	−2.49
ΔH	0.58	0.18	−0.96	−1.93	−1.81	−1.32	−1.94	−8.66	−9.37
J	1.53	0.31	3.84	1.33	−3.33	0.41	−0.83	0.82	−3.47
ΔJ	0.27	0.04	−6	−6.29	−6.64	−0.54	−1.51	−13.81	−15.19
K	1.32	0.09	−0.77	−1.59	−2.2	−0.54	−1.85	−1.9	−4.44
ΔK	0.06	0.06	−6.51	−6.6	−6.71	−1.01	−1.39	−13.2	−13.1
L	1.1	0.12	−2.26	−2.51	−3.44	−1.41	−1.7	−1.1	−3.13
ΔL	0.16	0.07	−4.55	−4.69	−4.66	−0.54	−1.37	−6.68	−6.65
Worked time									
B	1.49	0.2	−1.79	−0.93	−1.64	0.2	−1.64	−1.39	−4.83
ΔB	0.06	0.04	−8.24	−8.62	−8.57	−1.97	−2.27	−15.19	−15.1
C	1.46	0.18	1.56	−0.78	−1.1	−0.27	−1.51	−1.89	−5.5
ΔC	0.13	0.11	−9.72	−10.19	−10.19	−0.83	−1.27	−16.66	−16.76
D	1.4	0.29	−2.73	−4.44	−2.12	−0.75	−1.95	−2.2	−5.44
ΔD	0.13	0.09	−1.98	−2.52	−3.87	−1.96	−2.93	−19.91	−19.92
E	1.55	0.11	2.05	0.09	−2.94	0.66	−2.74	−3.11	−9.92
ΔE	0.25	0.16	−1.35	−2.02	−1.71	0.25	−1.32	−29.52	−30.27
F	0.42	0.21	−1.12	−3.42	−3.38	−2.31	−2.18	−3.55	−3.93
ΔF	0.14	0.14	−2.12	−2.29	−2.53	−0.16	−1.3	−16.13	−15.94
G	1.49	0.17	1.51	−1.12	−0.81	−0.57	−1.47	−1.89	−5.71
ΔG	0.2	0.15	−3.64	−3.99	−4.02	−1.09	−1.38	−16.79	−16.81
H	1.54	0.28	2.24	−0.02	−2.02	−0.27	−1.55	−1.12	−7.07
ΔH	0.24	0.17	−1.93	−2.88	−2.93	−1.39	−2.27	−32.75	−33.41
J	1.53	0.34	3.45	3.09	−0.83	−0.04	−1.2	0.21	−5.43
ΔJ	0.26	0.07	−0.06	−3.35	−7.79	0.57	−1.23	−22.94	−26.05
K	1.4	0.1	−1.02	−0.82	−4.6	0.59	−3.57	−4.95	−7.68
ΔK	0.09	0.07	−3.23	−3.45	−3.43	−0.3	−2.05	−21.76	−21.6
L	1.26	0.1	−3.48	−0.64	−3.93	−0.34	−1.76	−3.64	−6.54
ΔL	0.14	0.07	−3.57	−4.45	−4.4	−2.65	−3.34	−16.74	−17.01

Notes: The header presents critical values in round brackets.

Source: Author's own elaboration.

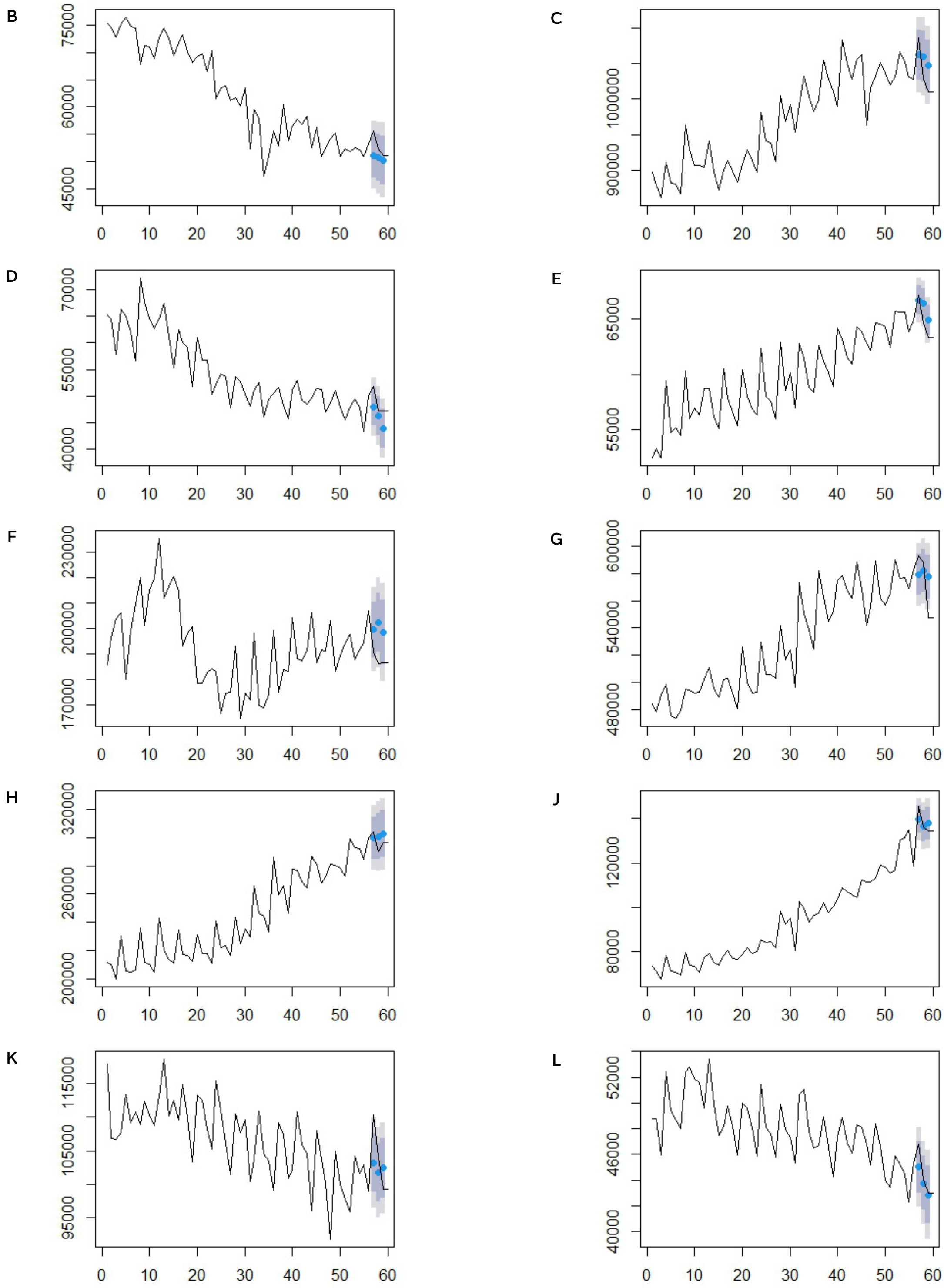
Figure 1A. Temporal disaggregation of annual time actually worked (in thousands of hours worked) to quarterly frequency using the Litterman method (with the average employment as the only high-frequency indicator series) across economic activity sections in Poland from Q1 2009 to Q4 2023



Notes: The temporal disaggregation in these plots is conducted using solely average employment as a high-frequency (quarterly) indicator series.

Source: Author's own elaboration.

Figure 2A. ARIMA-based forecasts of preliminary data on hours actually worked in Q1, Q2, and Q3 of 2023 compared to official survey data disseminated by Statistics Poland (Q1 2009 to Q4 2023)



Notes: ARIMA models were selected based on the Akaike Information Criterion, with an estimate of seasonal strength as per Wang et al. [2006]. Model selection followed the algorithm presented in Hyndman and Khandakar [2008]. The plots display point forecasts and confidence intervals (80% and 95%) shaded in grey, compared to the Official Survey Data from Statistics Poland, indicated by the black line. The ARIMA models chosen across economic activity sections are as follows: B: ARIMA (0,1,1) with drift; C, D: ARIMA (3,1,0) with drift; E, G: ARIMA (4,1,0) with drift; F: ARIMA (0,1,5); H, J: ARIMA (0,1,2) with drift; K: ARIMA (3,1,0); L: ARIMA (3,1,2).

Source: Author's own elaboration.

Table 5A. Supplementary auxiliary time series used in addition to average paid employment

Variable	Original description	Original data source	Adjustments applied in this study
Real Industrial Production Index (IPI)	Industrial Production Index (2015=100) for Poland; nominal values; unadjusted data (i.e., not seasonally or calendar adjusted), quarterly frequency.	OECD	Adjusted for inflation using the price index (implicit deflator; 2015=100) from Eurostat to obtain real values
Real GDP (GDP)	Gross domestic product at market prices for Poland (in millions of national currency); unadjusted data (i.e., not seasonally or calendar adjusted); ESA 2010 classification, quarterly frequency.	Eurostat	Converted to constant prices (2015=100) using the implicit deflator from Eurostat to derive real GDP
Real GDP *per capita* (GDP*pc*)	Gross domestic product at market prices per capita for Poland (in units of national currency); unadjusted data (i.e., not seasonally or calendar adjusted); ESA 2010 classification, quarterly frequency.	Eurostat	Converted to constant prices (2015=100) using the implicit deflator from Eurostat to obtain real *GDP per capita* (Poland)
Real Gross Value Added (GVA) for Poland and by section (Sections B–E, F, G–I, J, K, L*)	Gross value added at current prices in million euro (SCA) across sections, unadjusted data, quarterly frequency.	Eurostat	Converted to real GVA in million PLN (Poland) using the implicit deflator (2015=100) from Eurostat; adjusted for inflation and converted from EUR to PLN using quarterly average exchange rates from the National Bank of Poland

Notes: For the four primary auxiliary time series, alternative variants were also considered. These include versions lagged by one or two quarters, their seasonal components extracted using the X-13ARIMA-SEATS procedure (both contemporaneous and lagged values), as well as real indices adjusted by subtracting the corresponding seasonally adjusted series generated by X-13ARIMA-SEATS (also including contemporaneous and lagged values). Data on GVA by economic activity are available only from Q1 2021 onwards, while aggregate GVA data for Poland are available for the entire time span considered in this study.

Source: Author's own elaboration.

Table 6A. Mean Absolute Percentage Error (MAPE) of the Litterman-minrss method with selected auxiliary time series combined with the average paid employment series

Auxiliary time series	MAPE (%)									
	B	C	D	E	F	G	H	J	K	L
	Economic activity section									
average paid employment	2.65	1.83	3.85	2.48	2.65	1.98	3.52	3.35	2.97	2.68
Supplementary auxiliary time series used in addition to average paid employment:										
IPI	2.66	1.82	3.85	2.46	2.65	1.99	3.42	3.34	2.97	2.68
GDP	2.67	3.35	3.86	2.43	2.54	1.69	2.58	3.49	3.07	2.70
GDPpc	2.67	3.23	3.86	2.46	2.54	1.69	2.59	3.46	3.05	2.70
GVA	2.65	2.26	3.85	2.41	2.50	1.75	2.42	3.32	3.21	2.69
IPI[s]	2.66	23.53	3.79	2.48	3.46	1.96	15.56	3.32	3.17	3.97
IPI[u]	10.2	23.53	3.85	2.67	3.46	12.09	15.56	20.21	3.17	3.97
GDP[s]	5.46	13.87	9.46	25.66	20.91	13.11	14.33	24.09	4.88	8.05
GDP[u]	9.67	13.20	4.86	14.70	6.46	15.66	2.53	43.96	6.90	5.64
GDPpc[s]	179	143.74	32.05	70.84	37.32	13.24	105.70	73.33	37.12	40.69
GDPpc[u]	30.6	19.93	4.99	17.19	4.86	28.16	4.00	70.59	25.54	3.72
GVA[s]	15.1	24.85	6.64	3.51	14.71	19.76	30.14	60.18	21.29	2.73
GVA[u]	33.6	14.99	8.81	7.73	25.48	1.70	2.64	24.43	22.15	4.62
GVA_Section	2.65	1.85	3.86	2.47	2.65	2.00	3.55	3.40	2.98	2.68
IPI (-2)^{n}	2.64	1.85	3.84	2.47	2.66	1.99	3.60	3.20	2.94	2.65
GDP (-2)^{n}	2.72	2.00	3.79	2.45	2.66	2.40	4.10	2.78	2.91	2.61
GDPpc (-2)^{n}	2.72	1.99	3.78	2.43	2.66	2.25	4.09	2.78	2.90	2.61
GVA (-2)^{n}	2.73	2.32	3.79	2.49	2.66	2.38	3.79	2.82	2.95	2.60

cont. Table 6A

Auxiliary time series	MAPE (%)									
	B	C	D	E	F	G	H	J	K	L
	Economic activity section									
IPI[s] (-2)^n	2.66	1.83	3.85	2.48	2.65	1.98	3.51	3.29	2.96	2.68
IPI[u] (-1)^n	3.25	1.83	3.85	2.47	2.66	1.98	3.51	3.29	2.92	2.68
GDP[s] (-1)^n	2.44	2.44	4.13	1.82	2.96	2.69	4.38	6.91	2.76	2.39
GDP[u] (-2)^n	2.72	2.28	4.10	1.93	2.90	2.49	4.07	5.16	2.77	2.63
GDPpc[s] (-1)^n	2.38	2.29	4.15	1.69	2.79	2.50	3.66	8.47	2.77	2.24
GDPpc[u](-2)^n	2.72	2.19	4.15	1.76	2.71	2.39	3.49	5.78	2.83	2.78
GVA[s] (-1)^n	2.44	2.11	4.26	1.60	2.74	2.39	2.89	6.56	**2.75**	2.32
GVA[u] (-1)^n	2.41	2.05	4.21	1.64	2.69	2.30	2.97	4.87	2.75	2.38
IPI (-2)^a	2.64	1.85	3.84	2.47	2.66	1.99	3.51	3.45	2.94	2.67
GDP (-2)^a	2.82	2.09	3.77	2.47	2.66	2.26	3.81	2.79	2.92	2.62
GDPpc (-2)^a	2.82	2.06	3.76	2.46	2.66	2.26	3.81	2.80	2.92	2.63
GVA (-2)^a	2.82	2.62	3.77	2.47	2.65	2.30	4.02	2.86	2.97	2.62
IPI[s] (-2)^a	2.66	2.62	3.79	2.48	2.65	1.96	3.52	3.32	2.96	2.69
IPI[u] (-1)^a	3.25	1.83	3.85	2.47	2.66	1.98	3.51	3.29	2.92	2.68
GDP[s] (-1)^a	2.65	16.67	4.18	1.72	2.99	11.6	10.04	20.1	4.22	2.71
GDP[u] (-2)^a	5.30	14.33	4.37	1.84	3.01	6.99	9.20	14.2	5.07	2.67
GDPpc[s] (-1)^a	14.3	152.4	40.8	91.4	30.0	14.6	155.9	42.3	9.93	33.8
GDPpc[u] (-2)^a	43.6	35.91	4.36	2.11	5.03	3.44	4.39	3.36	10.4	3.25
GVA[s] (-1)^a	12.2	39.74	3.83	3.56	9.98	13.1	21.74	24.7	4.27	5.05
GVA[u] (-1)^a	2.75	27.62	4.31	2.75	10.2	10.4	14.37	37.0	3.99	6.27

Notes: [s] denotes the seasonal component of the given macroeconomic time series, extracted using the X-13ARIMA-SEATS procedure. [u] refers to the seasonally unadjusted real macroeconomic time series adjusted by subtracting the corresponding seasonally adjusted series generated using X-13ARIMA-SEATS. Lags of the macroeconomic time series are indicated in parentheses. The superscript *n* signifies that a naive forecasting approach was employed to extend lagged auxiliary time series to match the length of the original series in order to provide timely official statistics. The superscript *a* indicates that forecasts were generated using ARIMA models selected according to the Akaike Information Criterion (AIC), incorporating an estimate of seasonal strength following Wang et al. [2006]. Model selection in this case followed the algorithm proposed by Hyndman and Khandakar [2008]. GVA_Section refers to gross value added data disaggregated by specific economic activity sections (Sections B–E, F, G–I, J, K, and L). The selection between one-quarter and two-quarter lags for auxiliary time series was based on the criterion of minimising MAPE.

Source: Author's own elaboration.

Table 7A. Forecast accuracy across economic activity sections for Litterman temporal distribution of annual time actually worked (in thousands of hours) to quarterly frequency across economic activity sections in Poland from Q1 2009 to Q4 2019

Nace Rev. 2	ME	RMSE	MAE	MPE	MAPE	ACF1	Theil's U
B	−1.44E-10	2207.27	1582.03	−0.15	2.62	−0.51	0.54
C	3.98E-09	23461.77	18165.05	−0.06	1.87	−0.21	0.62
D	−5.30E-10	2791.18	2185.56	−0.26	3.91	−0.26	0.57
E	−8.88E-10	1412.02	1142.24	−0.07	1.95	−0.10	0.43
F	−5.23E-10	6886.16	5365.42	−0.17	2.82	−0.27	0.48
G	3.35E-09	10498.23	8247.70	−0.05	1.58	−0.23	0.46
H	−1.70E-08	8122.39	6283.94	−0.16	2.67	−0.29	0.42
J	−1.22E-07	3039.31	2279.77	−0.14	2.63	−0.34	0.48
K	−2.45E-11	3483.13	2826.16	−0.10	2.62	−0.35	0.59
L	−3.29E-10	1476.00	1183.76	−0.11	2.42	−0.15	0.56
average	0.00	6337.75	4926.16	−0.13	2.51	−0.27	0.51
s	0.00	6407.76	4967.58	0.06	0.61	0.11	0.07

Source: Author's own elaboration.

Temporal Disaggregation Models: From Chow–Lin to Dynamic Extensions

The Chow-Lin, Fernandez, and Litterman methods are three regression-based approaches that employ a Generalized Least Squares (GLS) regression of y_l, on the annualised quarterly indicator series, CX. These models assume a static and linear relationship between y and X, as well as between y_l and CX, leading to the expression $p = \hat{\beta}X$.

C is the temporal aggregation – extrapolation matrix: $I_{n_l} \otimes \omega$, which is the Kronecker product of an identity matrix of size n_l and a transposed vector ω of length $\frac{n}{n_l}$ (frequency conversion ratio) which specifies the form of temporal aggregation.

The Chow-Lin method assumes that the quarterly residuals adhere to a first-order autoregressive process (AR1), where $u_t = \rho u_{t-1} + \epsilon_t$ with ϵ_t being white noise: $\epsilon_t \sim N(0, \vartheta_\epsilon)$, and $|\rho| < 1$. This assumption leads to:

$$v(\rho) = \frac{\vartheta_\epsilon^2}{1-\rho^2} \cdot \begin{bmatrix} 1 & \rho & \dots & \rho^{n-1} \\ \rho & 1 & \dots & \rho^{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \dots & 1 \end{bmatrix} \qquad (1),$$

The distribution matrix D, a function of (v), is typically given by:

$$D(v) = vC'(CvC')^{-1} \qquad (2).$$

The GLS estimator $\hat{\beta}$, for a given variance-covariance matrix (v) is given by:

$$\hat{\beta}(v) = [(X'C'(CvC')^{-1}CX)]^{-1} X'C'(CvC')^{-1} y_l \qquad (3),$$

In contrast to the Chow-Lin method, the Fernandez and Litterman methods assume that the quarterly residuals follow a non-stationary process, represented as $u_t = u_{t-1} + v_t$, where v_t follows an autoregressive process of order 1. The Fernández method can be viewed as a special case of the Chow-Lin method when the quarterly innovation follows a near-random walk. Given its $I(1)$ properties, the Fernández approach implicitly assumes no cointegration between y and X, making its estimation process more stable [**Quilis, 2018**].

The Litterman method extends the Fernández method by modifying its $I(1)$ high-frequency innovation to an $ARI(1,1)$ process, introducing greater flexibility to capture highly trended processes. When $\rho = 0$, the Litterman model reduces to Fernández's model, making Fernandez its special case where u follows a random walk. The variance-covariance matrices for the Fernández and Litterman methods are given by equations (4) and (5) respectively:

$$v(0) = \vartheta_\epsilon^2 (G'G)^{-1} \qquad (4),$$

$$v(\rho) = \vartheta_\epsilon^2 (G'H'HG)^{-1} \qquad (5),$$

where G and H are $n \times n$ matrices. Both matrices have 1s along the main diagonal, represented by vector $\mathbf{1}_n$. In matrix G, the first subdiagonal consists of vector -1_{n-1}, whereas in matrix H, the entries of -1 are replaced by $-\rho$. All other entries in both G and H are zero.

Estimating the variance-covariance matrix involves determining ρ, which can be achieved through various methods, such as the iterative approach proposed by **Chow and Lin** [**1971**], or by minimising the weighted residual sum of squares (RSS), as introduced by **Barbone et al.** [**1981**]:

$$RSS(\rho) = u_l'(CvC')^{-1} u_l \qquad (6),$$

where u_l is the parameter obtained from the observed autocorrelation of the low-frequency residuals. As a result, the outcomes are dependent on the specification of (v) [**Sax, Steiner, 2013**].

Santos Silva and Cardoso [**2001**] introduced a dynamic extension of the classical Chow – Lin method. This extension incorporates a more complex dynamic structure into the relationship between y and X: $y_t = \varphi y_{t-1} + x_t \beta + \varepsilon_t$, $(-1 < \varphi < 1;\ t = 1,\ \ldots,\ n;\ \varepsilon_t$~$iddN(0,$ ~ iid N $(0, \vartheta_\varepsilon))$ where x_t represents the t-th row of X. The matrix form of the model is given as follows:

$$D_\varphi y = X\beta + q\eta + \varepsilon \tag{7}$$,

where D_φ: $n \times n = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 \\ -\varphi & 1 & 0 & \ldots & 0 & 0 \\ 0 & -\varphi & 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & -\varphi & 1 \end{bmatrix}$. Here, $q = \begin{bmatrix} \varphi \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\eta = E(y_0)$ represent the truncation remainders.

The (v) is given by $C\delta(\varphi)C'$, where $\delta(\varphi) = \vartheta_\alpha \left(D_\varphi D'_\varphi\right)^{-1}$, and ϑ_α is the standard deviation of the Gaussian white noise $a_t = iddN(0,\ \vartheta_\alpha)$ that linearly links X and y_l in the methods previously described. For further details, see **Quilis** [**2018**]. The estimation of y follows the same best linear unbiased estimator (BLUE) approach utilised in the Chow-Lin method. However, it is important to note that the Santos Silva and Cardoso method, while also assuming cointegration between y and X, does not fully incorporate the Chow-Lin procedure [**Quilis, 2018**].