GOSPODARKA NARODOWA

4(320)2024, 30–50
DOI: 10.33119/GN/190910
gnpje.sgh.waw.pl
SGH

The Polish Journal of Economics

Istnieje od / Published since 1931

Bartłomiej Wiśnicki
Department of Business Economics, SGH Warsaw School of Economics, Poland

Consumer Inertia and Dynamic Price Competition*

Inercja konsumentów a dynamiczna konkurencja cenowa

Keywords:
decision making, inertia, default effect, anecdotal reasoning

JEL classification codes:
D11, D91, L13

Article history:
submitted: February 9, 2024
revised: June 18, 2024
accepted: July 8, 2024

Abstract

The aim of this research is to study firms' decisions about price and quality in a setting where consumers cannot fully evaluate products and must rely on anecdotal evidence to make choices. Additionally, consumers exhibit inertia, or an overattachment to past purchases, in their decision making. To analyse firms' behaviour, we employ a 2-period price competition model in which firms can choose their product quality. Consumers are aware of prices but learn about product quality through experience and anecdotal evidence, which influence their choices according to a simple decision rule. We obtain analytical results in the form of a Nash equilibrium for pricing and quality strategies across various levels of consumer inertia and conduct comparative statics. Our findings suggest that inertia intuitively makes the market less competitive as firms gain monopoly power over attached consumers. However, inertia also intensifies competition for unattached consumers and encourages firms to enhance product quality. Consequently, in certain situations, inertia can reduce product prices and contribute to improved market welfare.

Streszczenie

Celem opisanych badań była analiza decyzji firm dotyczących ceny i jakości w warunkach, w których konsumenci nie mogą w pełni ocenić produktu, przy dokonywaniu wyboru muszą polegać na anegdotycznych dowodach oraz wykazują inercję (nadmierne przywiązanie do wcześniejszych zakupów) w swoim procesie decyzyjnym. Aby przeanalizować zachowanie firm, zastosowano dwuokresowy model konkurencji cenowej, w którym firmy mogą również wybierać jakość swoich produktów. Konsumenci są świadomi cen, ale uczą się jakości produktu poprzez doświadczenie i dowody anegdotyczne, które przejawiają się w prostych regułach decyzyjnych. Uzyskano analityczne wyniki w postaci równowagi Nasha w strategiach cenowych i dotyczących jakości dla różnych poziomów inercji konsumentów oraz przeprowadzono statykę porównawczą wyników. Zauważono, że zgodnie z przewidywaniami inercja czyni rynek mniej konkurencyjnym, ponieważ firmy zyskują monopolistyczną władzę nad przywiązanymi konsumentami. Inercja prowadzi jednak również do zwiększonej konkurencji o konsumentów nieprzywiązanych oraz do poprawy jakości produktów. W związku z tym w pewnych sytuacjach może przyczynić się do obniżki ceny produktu i tym samym do poprawy dobrobytu rynkowego.

Słowa kluczowe:
podejmowanie decyzji, inercja, efekt domyślny, anegdotyczne rozumowanie

Kody klasyfikacji JEL:
D11, D91, L13

Historia artykułu:
nadesłany: 9 lutego 2024 r.
poprawiony: 18 czerwca 2024 r.
zaakceptowany: 8 lipca 2024 r.

* Source of funding: National Science Centre, Poland, grant no. 2017/25/N/HS4/02331.

Introduction

Inertia is a widespread trend among consumers. Contrary to the predictions of neoclassical economics, decision makers tend to be biased by their previous choices and are more likely to stick with a brand than a rational model of choice would suggest. Empirical research provides substantial evidence that consumers remain attached to their choices. There are documented examples that empirically identify consumer inertia in markets such as employment insurance, cf. Osterman [1987]; digital imaging, cf. Tripsas and Gavetti [2000]; breakfast cereals; cf. Shum [2004]; health insurance, cf. Handel [2013]; and mortgage, cf. Andersen et al. [2015].

Consumer inertia can be linked to many aspects of boundedly rational decision-making, as explored in psychological and economic literature. Theoretical economic literature highlights switching and search costs, as noted by Farrell and Klemperer [2007]. These costs can be due to either rational optimisation or an agent-embodied fallacy, such as inattention, (cf. Ericson [2014]; Sitzia, Zheng, Zizzo [2015]) or status quo bias (cf. Samuelson and Zeckhauser [1988]. Inertia can also be associated with default effects (cf. DellaVigna [2009]), where a consumer's last purchase serves as a reference point for any market research. Similar insights come from other psychological phenomena as reflected by the endowment effect (cf. Ericson and Fuster [2011]) and the choice overload hypothesis (cf. Kamenica, Mullainathan, and Thaler [2011]).

Inertia has been described within the framework of Bewley's Knightian decision theory (cf. Bewley [1986]; Bewley [1987]). Inertia is placed in the decision-making framework as a natural assumption in the context of decision-making under uncertainty, where the assumption of the completeness of preferences is dropped. The inertia assumption suggests that, in cases of indecision, a consumer will default to the last purchased option rather than switching to a new one. This reasoning aligns with Simon's concept of satisficing behaviour [1955]. Further work by Masatlioglu and Ok [2005], Sagi [2006], Ortoleva [2010], and Masatlioglu and Ok [2014] also rationalises status quo bias and the endowment effect within decision theory. These contributions share a key principle: a consumer provided with a satisfactory product is less likely to switch to another for an uncertain reward.

This research aims to examine the consequences of consumer inertia in a duopoly market, focusing on how this friction impacts firms' strategic decisions and, subsequently, market welfare.

To incorporate inertia from decision theory into a market framework, several adjustments are needed. First, as the literature suggests, modelling consumer inertia necessitates some uncertainty about product quality or performance. For this purpose, we use Spiegler's [2006b] framework of anecdotal reasoning. Here, consumers follow the $S(1)$ procedure of Osborne and Rubinstein [1998] to determine if a product meets their needs and provides positive utility. The procedure can be described as follows: a consumer learns a piece of information about each available product, such as a recommendation or lack thereof, and then selects the cheapest product with a positive recommendation.

We extend this choice procedure by adding inertia: now, the consumer first examines whether the last purchased product was satisfactory based on personal experience. If it was, then with some probability, she will buy it again without further market research. This rule combines inertia with imperfect product comparison, as suggested by the Knightian Decision Theory. It also links inertia to uncertain product quality as it is suggested by empirical evidence, cf. Crawford, Tosini, and Waehrer [2011], Goettler and Clay [2011], Grubb and Osborne [2015].

We apply the decision procedure in a dynamic Bertrand-type duopoly model to analyse the firms' decision-making process in a market with consumer inertia. First, firms decide on the quality level of their products. Then, firms play a 2-period pricing game, competing for customers who follow the decision rule outlined above.

There is a major difference between the two pricing stages of the game. In the first stage, we assume that consumers have not yet used the product, so they do not exhibit inertia. However, in the second stage, they display inertia based on their first-period choices. Introducing this dynamic allows us to analyse period pricing strategies across periods: firms initially compete for consumers without inertia, and in the following period, they compete with a given share of attached consumers.

This research has several objectives. First, we aim to establish the pricing strategies firms adopt across periods in a market where consumers exhibit inertia. A typical strategy observed in the search and switching cost literature is a "harvest-invest" approach (cf. **Farrell and Shapiro** [**1988**], **Farrell and Klemperer** [**2007**]). In this approach, firms initially lower prices to attract new consumers, then raise prices to maximise profits in the subsequent period. We seek to investigate how inertia affects this behaviour and whether the gains from low first-period pricing offset the potential for higher pricing in the second period. Such dynamics cannot be analysed in a static model (cf. **Spiegler** [**2006**], **Szech** [**2011**], **Wisnicki** [**2022**]). Moreover, unlike **Wisnicki** [**2022**] where steady-state, long-term competition is analysed, this paper focuses on short-term between-period pricing competition, investigating market interactions on a period-by-period basis instead of zeroing in on long-term gains.

Another objective is to explore how inertia impacts firms' decisions about quality and how it affects market welfare. **Szech** [**2011**] argues that firms may not always set the highest possible quality levels when dealing with consumers who employ anecdotal reasoning, as high quality can intensify competition and drive prices down. Inertia may mitigate this effect by allowing firms to cultivate consumer loyalty, potentially promoting higher product quality. **Wisnicki** [**2022**] observed this outcome in a single-period model, similar to that in **Spiegler** [**2006b**] and **Szech** [**2011**]. However, introducing a dynamic competition model could significantly influence firms' quality decisions, as long-term gains from repeated purchases become more valuable.

The article is structured as follows: Section 2 outlines the consumer decision rule. Section 3 incorporates the decision procedure into a model of duopolistic price competition. Section 4 presents comparative statics on inertia, and Section 5 offers concluding remarks.

Consumer's decision rule

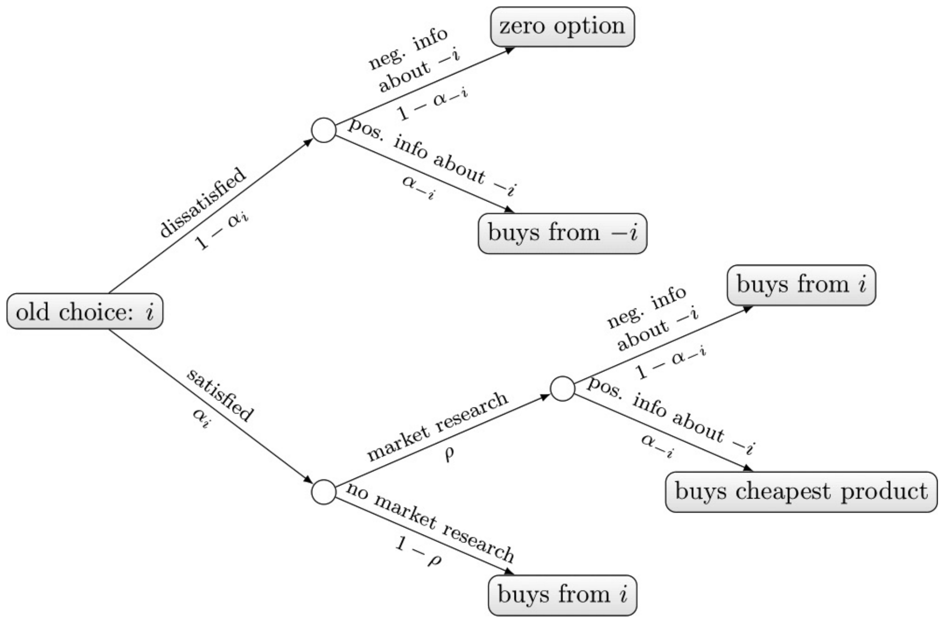
The decision rule is an extension of the anecdotal reasoning procedure proposed by **Spiegler** [**2006b**]. The extension adds inertia and describes the decision rule in two-period decision making. While the first part is thoroughly described in **Wisnicki** [**2022**], the second one is new.

We assume that there are multiple producers, each selling a single substitute good. Each product has a certain probability of providing satisfaction to the customer. Specifically, with probability $\alpha_i \in [0,1]$, a product sold by firm *i* provides the consumer with positive utility.

There are two decision-making periods, $t = 1, 2$. In the first period, consumers follow a simple *S*(1) procedure of **Osborne and Rubinstein** [**1998**]. They obtain a single piece of information about each product, which is in the form of a binary signal: the product of firm *i* can either be "good" or "bad". The probability of a positive signal is $\alpha_i \in (0,1]$ for firm *i*. The consumer then chooses the cheapest product that she received a positive anecdote about. If a consumer does not receive any positive signal, she chooses a costless outside option (hence refrains from choosing).

In the second period, consumers exhibit inertia. Thus, before the above procedure occurs, they evaluate their last chosen product. The result of this evaluation is also binary and has the same probability of being positive as in the first period (α_i for a firm *i*). Then, with probability $1-\rho$, she remains with her last choice and buys the new product from the same firm. However, suppose the consumer is dissatisfied with his or her last purchased product or is satisfied with probability ρ. In that case, she obtains information about the competitor's product regarding its fit for the consumer and decides similarly to the first-period procedure. The procedure for the second period is presented in Figure 1. This graph can also be used to describe the simpler first-period procedure: we need to impose $\rho = 1$ (so no inertia present).

Figure 1. Decision procedure for a consumer in the second period that owns product *i* in the duopoly market



Note: $-i$ represents the competitor of i.
Source: Figure originally in Wisnicki [2022].

The described decision rule is based on two groups of parameters. First, the value of α_i can be associated with the product's technical quality or suitability to a single consumer. Similar to Spiegler [2006b], α_i is the probability of giving the consumer a positive utility and (by extension) the probability of receiving a positive anecdote. Hence, the own experience and the anecdotal one are the same information signals.

The value of $1-\rho$ provides us with information about the magnitude of consumer inertia. For $\rho = 1$, the consumers do not exhibit inertia, thus they evaluate each alternative signal as in Spiegler [2006b]. For $\rho \to 0$, consumers are highly attached to the previous purchase: if it was deemed satisfactory, they repeat the purchase ignoring the price difference.

In order to transfer from the individual consumer decision-making to firms' market shares, which is useful in the game presented in the following section, we assume that there is a unit mass of identical consumers, each endowed with the exact reservation price normalised to 1[1]. The same mass makes first- and second-period decisions. For simplicity, we assume no inflow or outflow of consumers. Hence firms operate on fractions of market shares rather than the number of consumers.

Using market shares and assuming that only previous purchases impact consumer decisions (rather than other past actions) allows us to model market share transfers as Markov chains. The switching probability for an individual consumer depends on product quality, the level of inertia, and prices. However, note that it is the order of prices that matters, rather than the specific values.

Model of duopoly

We now analyse a 2-period model of duopoly price competition with consumer inertia[2]. We assume that there is a continuum of consumers defined on a unit interval. At each time $t = 1,2$, every consumer independently makes a choice of option $i = 0,1,2$, where $i = 1,2$, denote products of firms 1 and 2 respectively, while $i = 0$ is the costless zero option of not choosing any product. Each period's decision is made according to the decision rule illustrated in Section 2. However, the decision process is different in each period. We assume

1 This assumption is equivalent to stating that each consumer gets a utility of 1 upon finding a satisfying option, as the net expected consumer gain from purchasing option i is $\alpha_i - p_i$.

2 The model is part of an unpublished PhD thesis by Wisnicki [2023].

that consumers are inactive in the first period and so they do not exhibit inertia. Conversely, in the second period, they are influenced by their past choice, so they decide to follow the decision rule[3].

We can envision the following situation modelled by this procedure. Firms offer products that change or are frequently updated (such as cars, mobile phones, or laptops). Consumers need to buy a new product at each period. At $t=1$, consumers are inexperienced and decide which brand to purchase based on anecdotal information. In the second period, consumers first determine whether their experience with the last-purchased product was satisfactory. If it met their expectations, they may repurchase from the same brand without further market research. For simplicity, we assume that no additional purchases occur after the second period. The consumer pool is also fixed, meaning that the only inactive consumers at $t=2$ are those who opted for the zero option in the previous period.

Firms compete on price over these two periods; that is, they independently and simultaneously choose prices $p_{i,t}$ for firm *i* in each period *t*. We assume that firms possess full information about the market, consumer behaviour, and competitors' products. They also know the price strategies in the game's previous stages. Firms can supply the whole market and incur constant marginal production costs normalised to zero.

Before the price decision, firms choose the quality of their product, represented by the parameter α_i, which reflects the probability of consumers being satisfied with the product, and thus the chances of other consumers receiving positive information about it. A firm can choose any quality parameter in the unit interval and does so costlessly. The chosen quality cannot be changed between periods, and its true value is not known to the consumers; as a result, the firms decide using the described behavioural procedure.

In summary, the game has the following dynamics:

1. Firms decide on the quality levels $\alpha_i \in [0,1]$,
2. Firms choose first-period prices $p_{i,1} \in [0,1]$,
3. Consumers perform decision rule without inertia,
4. Firms choose second-period prices $p_{i,2} \in [0,1]$,
5. Consumers perform decision rule with inertia.

After points 2 and 4, firms receive market shares and profits. Thus, the model can be represented by a dynamic, two-period game where each period is represented by firms choosing quality levels and consumers making their purchase decisions. Therefore, the solution of the model is a subgame perfect Nash equilibrium represented by a tuple $(\boldsymbol{\alpha}_i, \mathbf{p}_{i,t})$, where $\boldsymbol{\alpha}_i$ is a two-element vector of quality parameters and $\mathbf{p}_{i,t}$ is a 2×2 matrix of prices chosen by firm *i* at time *t*.

When consumers do not exhibit inertia and have full information about product quality, firms choose the best possible quality and the lowest prices in both periods. This scenario resembles a finitely repeated Bertrand competition with symmetric firms, maximising consumer surplus and leaving firms with no profits. Therefore, any positive profits for firms and the existence of inferior products arise from behavioural friction characteristic of consumers.

Price competition in the second period

We solve the model by backward induction. We first find the firms' strategies in the second period (assuming some chosen quality and market share division after the first-period competition). Then we sequentially analyse the pricing strategy in the first period and quality setting stage.

After the first period, firms also know (and cannot change) their quality, chosen before the pricing stage. We assume, without loss in generality, that $\alpha_1 \geq \alpha_2$. We keep this assumption throughout the whole price equilibrium analysis.

---

3 Formally, consumers also decide based on the decision rule in the first period. However, as their previous choice is the zero option, inertia does not affect their decision procedure. Also, note that if a consumer did not find any satisfactory product and chose the zero option in the first period, she will not be affected by inertia.

Denote the opponent of firm i by $-i$. Assuming $p_{i,2} < p_{-i,2}$, the transition matrix of a single consumer in the second period is given by

$$Q_2 = \begin{pmatrix} (1-\alpha_i)(1-\alpha_{-i}) & \alpha_i & (1-\alpha_i)\alpha_{-i} \\ (1-\alpha_i)(1-\alpha_{-i}) & \alpha_i & (1-\alpha_i)\alpha_{-i} \\ (1-\alpha_i)(1-\alpha_{-i}) & \alpha_i(1-\alpha_{-i})+\alpha_i\alpha_{-i}\rho & (1-\alpha_i)\alpha_{-i}+\alpha_i\alpha_{-i}(1-\rho) \end{pmatrix}. \quad (1)$$

The consecutive rows and columns of Q represent the zero option, firm i, and firm $-i$, respectively. We refer to consumers represented by the last two rows of Q as "attached" to firm i and $-i$, respectively. The transition matrix is symmetric to the one in (1) if $p_{i,2} > p_{-i,2}$ (we disregard here the case where prices are equal: the transition matrix can be found in the Appendix). The transition matrix from (1) can be applied to every consumer, giving us market share transitions for a given relation of prices.

Following Armstrong and Vickers [2019], we know that there is a unique price equilibrium in which firms play mixed strategies defined on a common interval $[L,H]$, where $L,H \in [0,1]$. As $p=1$ is the monopoly price, we can deduce that $H=1$. The firms' strategies are continuous on this interval, with the possibility of a single firm playing an atom at $p=1$.

Which firm plays the atom depends strongly on their market shares after the first period. Let x_1 represent the market share attained by firm 1 after the first period. From matrix (1), we can deduce that the inactive portion of the market is given by $x_0 = (1-\alpha_1)(1-\alpha_2)$, regardless of the price levels. Therefore, we can describe the first-period market shares using only one variable, $x_1 \in [0, 1-(1-\alpha_1)(1-\alpha_2)]$. The following proposition outlines the conditions that distinguish between these two qualitatively different strategies.

Proposition 1. *Assume* $\rho < 1$. *Define* $x^* = \frac{1}{2}\left(\frac{1}{\alpha_1(1-\rho)} - \frac{1}{\alpha_2(1-\rho)} + \alpha_1 + \alpha_2 - \alpha_1\alpha_2\right)$. *In the unique equilibrium of the-second period price competition, if* $x_1 > x^*$, *then only the strategy of firm 1 has an atom at* $p=1$. *Conversely, if* $x_1 < x^*$, *then only firm 2 has an atom at* $p=1$. *If* $x_1 = x^*$, *no firm plays an atom at* $p=1$.

Proposition 1 states that the firm with a sufficiently high market share – given certain quality and inertia levels – sets an atom at $p=1$ in the equilibrium of the second-period price competition. Thus, an atom at $p=1$ can be associated with a better market position at the beginning of the second period. We call the atom-setting firm "prominent" because it holds an advantage over the opponent at the start of the second period. Intuitively, the firm with a higher market share would be more inclined to command a higher price for its product. The threshold of x^* is defined only when inertia is present. Without inertia (i.e. $\rho = 1$), consumers are not attached, and the prominent firm is the one with higher quality. Below is the formal presentation of equilibrium pricing strategies at $t=2$.

Proposition 2. *Assume that* $x_1 \geq x^*$, *where* x^* *is defined as in Proposition 1. Then, in the equilibrium of the second-period pricing stage, firms play strategies according to CDFs* $F_1(p)$ *and* $F_2(p)$ *of the following form:*

$$F_1(p) = \frac{\gamma_1}{\beta_1}\left(1 + \frac{\gamma_2}{(\beta_2-\gamma_2)p}\right),$$

$$F_2(p) = \frac{\beta_2-\gamma_2}{\beta_2} + \frac{\gamma_2}{\beta_2 p},$$

where

$\beta_1 = \alpha_1\left(\alpha_1(1-\rho)(1-\alpha_2) + (1-\rho)\alpha_2 - 1\right),$

$\beta_2 = \alpha_2\left(\alpha_1(1-\rho)(1-\alpha_2) + (1-\rho)\alpha_2 - 1\right),$

$\gamma_1 = 1-(1-\rho)\alpha_1 x_1,$

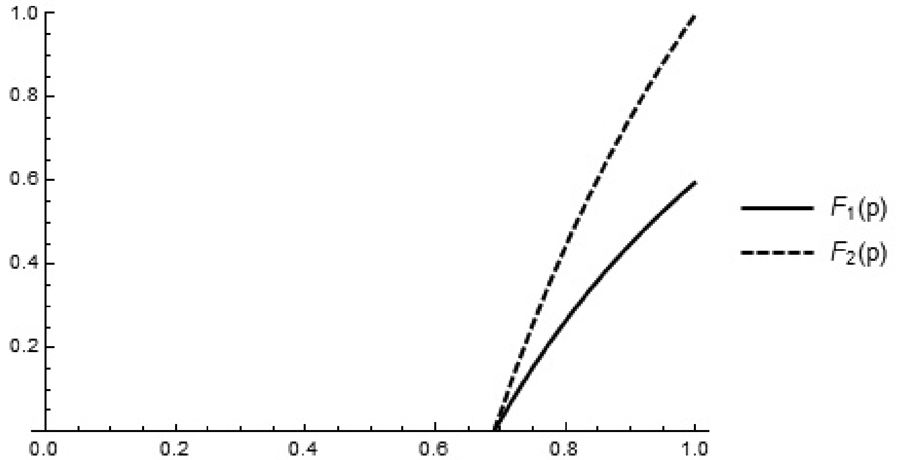
$\gamma_2 = 1+(1-\rho)\alpha_2 x_1 - \alpha_2.$

The support of strategies is the interval $[L,1]$, *where* $L=\frac{\gamma_2}{\gamma_2-\beta_2}$. *Firm 1 also plays an atom at* $p=1$ *in the size of* $\mu=1-\frac{\gamma_1}{\beta_1}\left(1+\frac{\gamma_2}{\beta_2-\gamma_2}\right)$. *In the case when* $x_1<x^*$, *the equilibrium is symmetric to the one above, with firm 2 setting an atom at* $p=1$. *The firms' equilibrium profits are*

$$\begin{aligned}\pi_{1,2}&=\alpha_1\left(1-\alpha_2\left(1-(1-\rho)x_1\right)\right),\\ \pi_{2,2}&=\frac{\alpha_2\left(1-\alpha_1(1-\rho)x_1\right)\left(\alpha_2\left(1-(1-\rho)x_1\right)-1\right)}{\alpha_2(1-\rho)\left(\alpha_1\left(1-\alpha_2\right)+\alpha_2-x_1\right)-1}.\end{aligned} \tag{3}$$

In the second stage of price competition, firms play mixed strategies, defined as continuous CDFs with a possible atom at $p=1$ set by one firm. $F_i(p)$ are concave functions defined on the interval $[L,1]$, where L is strictly positive. Thus, firms obtain positive profits compared to classic Bertrand price competition. Examples of equilibrium CDFs, for a given parametrisation, are depicted in Figure 2. Note that the firms' strategies and payoffs are asymmetric. This is because the parametrisation of the game in the second period is not symmetric: the firms' market shares after the first period are represented solely by the market share of firm 1.

Figure 2. Second-period price strategies in equilibrium for $\alpha_1=3/4$, $\alpha_2=1/2$, $\rho=1/2$, $x_1=1/2$



Source: Author's own elaboration.

Price competition in the first period

The first-period price competition is significantly distinct from that analysed in the previous section. First, firms' pricing decisions transfer as market shares into the second period and influence the subsequent price competition. Second, consumers behave differently in the two periods: at $t=1$, they do not exhibit inertia, as they are not initially attached to any brand.

The consumer choice in the first period depends solely on the order of prices and the outcome of product sampling. Thus, for the given prices p_i and p_{-i}, the market share of firm i is:

$$x_i=\begin{cases}\alpha_i & \text{if } p_i<p_{-i},\\ \alpha_i\left(1-\alpha_{-i}\right)+\alpha_i\alpha_{-i}/2 & \text{if } p_i=p_{-i},\\ \alpha_i\left(1-\alpha_{-i}\right) & \text{if } p_i>p_{-i}\end{cases}$$

Firms optimise their first-period price decisions, taking into account the sum of payoffs from both periods. The outcome of the first-period competition is transferred to the second period through variable x_1, indicating the market share of firm 1 after the first period and corresponding to its size of the set of attached consumers. Note that the second-period payoff is not differentiable around x^* so we need to study a specific case: one in which firm 1 is assured prominence in the second period, regardless of first-period choices. For a given set of qualities, the lowest market share that firm 1 can achieve is $\alpha_1(1-\alpha_2)$. If this value is higher than x^*, as defined in Proposition 1, that is, if the following conditions are satisfied:

$$(1+\alpha_1)\alpha_2 \le \alpha_1 \text{ or } \frac{\alpha_2-\alpha_1(1-\alpha_2)((1+\alpha_1)\alpha_2+1)}{\alpha_1\alpha_2(\alpha_2-\alpha_1(1-\alpha_2))} < \rho, \tag{5}$$

firm 1 is prominent regardless of the market share after the first period. Thus, firm 1 can ensure its prominence if its product quality is relatively high compared to its opponent. The quality gap may not be very substantial if inertia is moderate, as the second inequality of (5) suggests.

If the inequalities (5) are satisfied, the firms' strategies qualitatively resemble those in the second period. Following **Armstrong and Vickers [2019]**, firms again play mixed strategies defined on the interval $[L,1]$, $L \ge 0$. However, in contrast to the second period, putting some probability on $p=0$ in the firm's strategy may be beneficial, as it can lead to a higher market share before the second period. Following the language of **Farrell and Klemperer [2007]**, this behaviour is called "invest-harvest": firms lower their prices for new consumers only to exploit them in the subsequent periods. Moreover, due to the invest-harvest strategy, it is not necessarily straightforward which firm would play an atom at $p=1$. The following proposition provides the exact formulas for firms' strategies in the first period (recall that we assume $\alpha_1 \ge \alpha_2$).

Proposition 3. *Assume that inequalities (5) are satisfied. Then, in the unique equilibrium of the first-period price competition, firms play mixed strategies defined on the interval* $[J, 1]$, *where* $J = \max\{J_1, J_2, 0\}$ *and*

$$J_1 = 1-\alpha_1 + \frac{(1-\alpha_2)(1-\alpha_1^2(1-\alpha_2)(1-\rho))(1+\alpha_1\alpha_2(1-\rho))}{1+\alpha_2^2(1-\rho)} + \frac{(1-\alpha_1^2(1-\rho))(1-\alpha_2(1-\alpha_1(1-\rho)))}{1-(1-\alpha_1)\alpha_2^2(1-\rho)},$$

$$J_2 = 1-\alpha_2(1+\alpha_1\alpha_2(1-\rho)).$$

The strategies take the following form:

- *If* $J_1 \le J_2$, *then firms play strategies defined by CDFs* $G_1(p)$ *and* $G_2(p)$ *such that*
  - $G_1(p)$ *solves* $\pi_2^{NP}(p_2 = J) = \pi_2^{NP}(p_2 = p, G_1(p))$,
  - $G_2(p) = \frac{p-J}{1-(1-p)\alpha_2 - J}$, *which solves* $\pi_1^P(p_1 = 1) = \pi_1^P(p_1 = p, G_2(p))$,

  with firm 1 setting an atom of size $1-G_1(1^-)$ *at* $p=1$.
- *If* $J_1 > J_2$, *then firms play strategies defined by CDFs* $G_1(p)$ *and* $G_2(p)$ *such that*
  - $G_1(p)$ *solves* $\pi_2^{NP}(p_2 = 1) = \pi_2^{NP}(p_2 = p, G_1(p))$
  - $G_2(p) = \frac{p-J}{1-(1-p)\alpha_2 - J}$, *which solves* $\pi_1^P(p_1 = J) = \pi_1^P(p_1 = p, G_2(p))$,

  with firm 2 setting an atom of size $1-G_2(1^-)$ *at* $p=1$.

In addition, if $J<0$, *then both firms play an atom of size* $G_i(0)$ *at* $p=0$.

The total firms' profits in the equilibrium are then

$$\pi_1^P = \begin{cases} \alpha_1(1-\alpha_2)(\alpha_1\alpha_2(1-\rho)+2) & \text{if } J_1 \le J_2, \\ \alpha_1(1-\alpha_2)(\alpha_1\alpha_2(1-\rho)+2) & \text{if } J_2 < J_1, \end{cases}$$

$$\pi_2^{NP} = \begin{cases} \alpha_2\left(J + \frac{(1-\alpha_2)(1-\alpha_1^2(1-\alpha_2)(1-\rho))(1+\alpha_1\alpha_2(1-\rho))}{1-\alpha_2^2(1-\rho)}\right) & \text{if } J_1 \le J_2, \\ \alpha_2\left(1-\alpha_1 + \frac{(1-\alpha_1^2(1-\rho))(1-\alpha_2(1-\alpha_1(1-\rho)))}{1-(1-\alpha_1)\alpha_2^2(1-\rho)}\right) & \text{if } J_2 < J_1. \end{cases}$$

Continuous CDFs represent the equilibrium strategies defined on the interval $\left[J,1\right]$, where $J \geq 0$. We observe qualitatively similar strategies in both periods for a vast spectrum of parameter values: the lower bound of the strategies' support is strictly higher than zero. The only atom is played at $p=1$ by a single firm. At $t=1$, the firm that plays the atom is characterised by higher quality. Note that, although the game is symmetric in its construction, the firms' strategies and payoffs are not. This is due to the fact that Proposition 3 assumes inequalities (5), which make the game asymmetric in the firms' characteristics.

Proposition 3 does not present each equilibrium strategy explicitly. This is due to the calculation limitations of finding the exact formula for a general set of parameters. Therefore, we present the constructive formula for each strategy that equates a fixed payoff (at lower or upper bound of the equilibrium strategy) with a general payoff for any pure strategy inside the equilibrium support. The formulas give a single solution and thus a unique equilibrium profile of strategies for a given parametrisation.

Let us move to examining the situation in which firm 1 cannot guarantee its prominence in the first period; so inequalities (5) do not hold. A firm's payoff in the second period, if it plays some pure price strategy at $t=1$, is not smooth in x_1, which modifies the analysis. Despite this difference, the equilibrium strategy's characteristics should not differ from those in Proposition 3. If firms set positive prices, they should use continuous strategies, allowing for the possibility of one firm playing an atom at $p=1$ and both firms playing atoms at $p=0$.

The following proposition characterises both equilibrium scenarios (pure and mixed strategies):

Proposition 4. *Assume that inequalities (5) are not satisfied and $\alpha_1 \geq \alpha_2$.*

The unique equilibrium of the first-period price competition is characterised by the following profile of strategies:

If $\pi_1^0 > \pi_1^{NP}\left(p_{1,1}=1, p_{2,1}=0\right)$, then the equilibrium strategies are $p_{1,1}=0,\ p_{2,1}=0$.

If $\pi_1^0 < \pi_1^{NP}\left(p_{1,1}=1, p_{2,1}=0\right)$, then firms play mixed strategies defined on the interval $\left[K,0\right]$, where $K=\max\left\{K_1,K_2,0\right\}$ and

$$K_i = \frac{(1-\alpha_i)\alpha_i\alpha_{-i}^3(1-\rho)^2 + \alpha_{-i}\left(\alpha_i^2(1-\rho)\left(1-\alpha_i(1-\rho)-1\right)-1\right) - \alpha_i^2(1-\rho)+1}{1-(1-\alpha_i)\alpha_{-i}^2(1-\rho)},$$

for $i \in \left\{1,2\right\}$. The strategies are then of the following form:

Label firms so that $K_i \leq K_{-i}$. Then firms play mixed strategies $G_i(p)$ and $G_{-i}(p)$ such that

$$G_i(p) \text{ solves} \begin{cases} \pi_{-i}^P\left(p_{j,1}=K\right) \text{ for } p \geq p_i^*, \\ \pi_{-i}^P\left(p, G_i(p)\right) = \pi_{-i}^P\left(p_{j,1}=K\right) \text{ for } p < p_i^*, \end{cases}$$

$$G_{-i}(p) \text{ solves} \begin{cases} \pi_i^{NP}\left(p, G_{-i}(p)\right) = \pi_i^{NP}\left(p_{i,1}=1\right) \text{for} p \geq p_{-1}^*, \\ \pi_i^P\left(p, G_{-i}(p)\right) = \pi_i^{NP}\left(p_{i,1}=1\right) \text{for } p < p_{-1}^*, \end{cases}$$

where $$\begin{aligned} &p_i^* \text{ solves } \pi_{-i}\left(p_i^*, G_i^*\right) = \pi_{-i}^P\left(p_{j,1}=K\right), \\ &p_{-i}^* \text{ solves } \pi_i\left(p_{-i}^*, G_{-i}^*\right) = \pi_i^{NP}\left(p_{i,1}=1\right). \end{aligned}$$

If $K_i < K_{-i}$, then firm i sets an atom at $p=1$ of size $1-G_i(1)$. If $\max\left\{K_1,K_2\right\}<0$ then firms play an atom at $p=0$ of size $G_i(0)$ and $G_{-i}(0)$ for firms i and –i, respectively.

Firms obtain the following profits in equilibrium (summing up both periods):

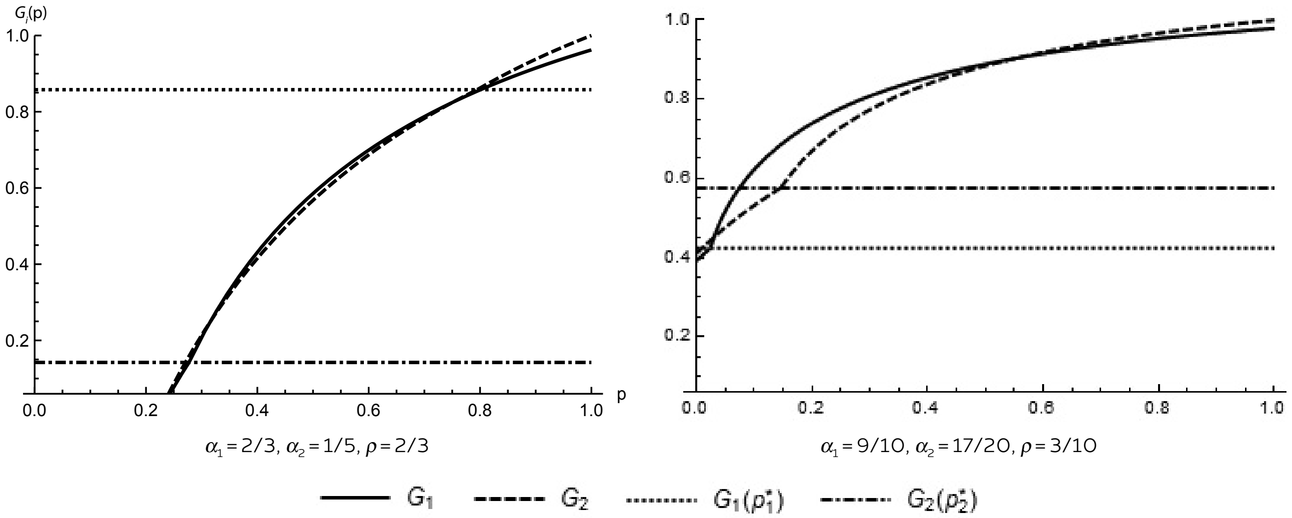
$$\pi_i = \pi_i^{NP}\left(p_{i,1}=1\right),$$
$$\pi_{-i} = \pi_{-i}^P\left(p_{j,1}=K\right).$$

If $\pi_1^0 = \pi_1^{NP}\left(p_{1,1}=1, p_{2,1}=0\right)$, then the equilibrium set includes both of the strategy profiles described above.

The equilibrium strategies of Proposition 4 share some similarities with those in Propositions 2 and 3. First, across a wide range of parameters, firms in equilibrium play a continuous strategy, with the support being a subset of the unit interval. Moreover, as in the case of Proposition 3, at most one firm plays an atom at $p=1$, and both firms may play atoms at $p=0$.

A key difference between the strategies in Propositions 3 and 4 is that the equilibrium strategy in Proposition 4 is not differentiable around p_i^* and is therefore a piecewise function. This is because firm 1 cannot guarantee its prominence in period 2. Thus, the continuation payoff is not smooth with respect to prices. Examples of such strategies (both with and without an atom at $p=0$) are depicted in Figure 3.

Figure 3. First-period equilibrium strategies for different parameter values

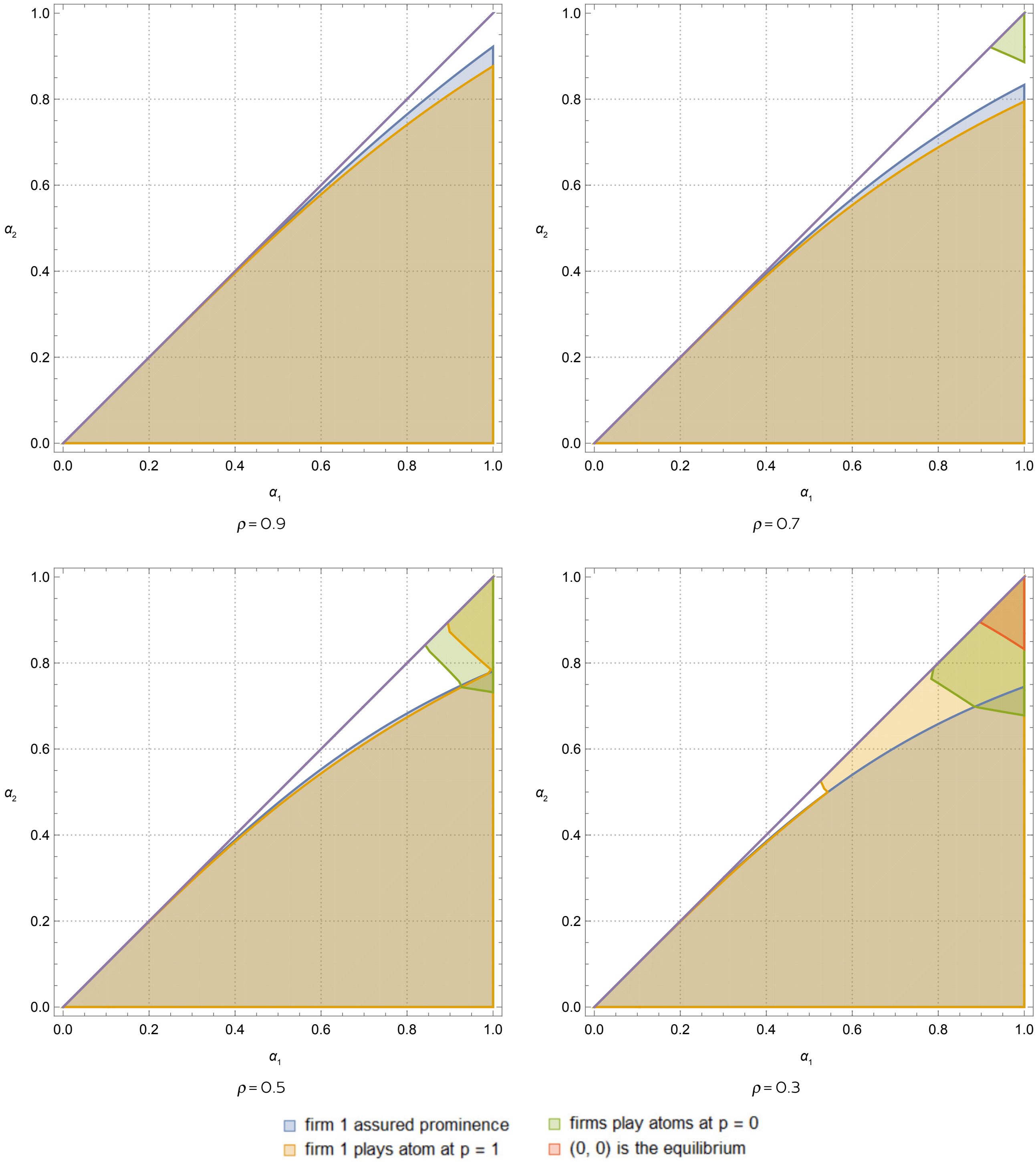


Source: Author's own elaboration.

For certain parameter ranges, firms set prices equal to 0 in the first period, gaining profits only through first-period price competition. This extreme strategy occurs when both quality and inertia are high, forcing firms into intense price competition due to a small number of transfers in the second period.

The type of strategy played in the equilibrium strongly depends on the model parameters, especially the consumer inertia parameter ρ. Figure 4 presents the regions of quality values for which the equilibrium has specific characteristics, having an inertia parameter fixed at various levels. In summary, there are at least three qualitative dimensions in which we can characterise the first period's equilibrium strategies. First, we determine whether firm 1 can ensure its prominence in the second period for any first-period strategy. This distinction is represented by inequalities (5) and distinguishes results in Propositions 3 and 4. Second, at most one firm plays an atom at $p=1$; it is firm 1 if $J_1 < J_2$ or $K_1 < K_2$, depending on the previous division. Third, firms may set an atom at $p=0$; and this occurs if $\max\{J_1, J_2\} < 0$ or $\max\{K_1, K_2\} < 0$, depending on the prominence assurance of firm 1. In addition, the equilibrium strategy may even be to forgo first-period profits and play $p=0$ as a pure strategy. This strategy occurs if the profit of firm 1, summed across the periods when both firms play $p=0$, exceeds the profit achieved by deviating to any other strategy at $t=1$.

Figure 4. Regions of quality values α_1 and α_2 for which the equilibrium follows specific characteristics (detailed in the legend)



Source: Author's own elaboration.

Quality equilibrium

Knowing the equilibrium forms of the two-stage price competition, we can now analyse the game's first step: the firms' decision concerning their quality levels. Let us recall that firms choose the quality of their products independently and costlessly; that is, each firm picks quality level $\alpha_i \in [0,1]$ for $i = 1,2$. These quality levels cannot be changed throughout the game. Moreover, we focus only on pure strategies in quality.

One intuitive notion suggests that, since quality is costless, firms would choose the maximum quality level. Higher quality means more returning customers and incentivises higher pricing. On the other hand, if both firms offer a high-quality product, price competition in both periods may result in low profits. Therefore, an alternative equilibrium candidate would be an asymmetric one, in which only one firm plays $\alpha_i = 1$, and the other settles for inferior quality. The significance of the asymmetric solution is greater if the level of inertia is low: the loss in the share of unattached consumers from the first period is not as significant to force a high-quality decision at the expense of greater price competition.

These insights, together with previous results, guide our search for equilibrium quality levels, as outlined in Proposition 5.

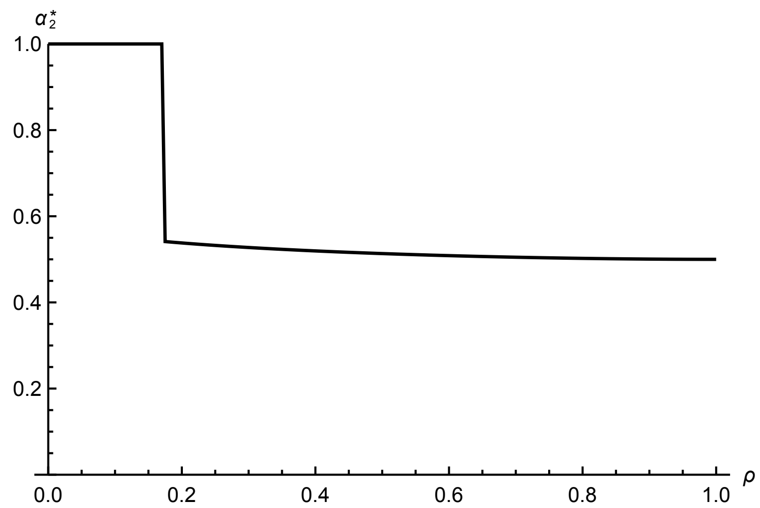
Proposition 5. *Assume, without a loss in generality, that* $\alpha_1 \geq \alpha_2$. *Let us define the value of* $\bar{\rho}$ *as a unique solution to* $\pi_2^{NP}(\alpha_1 = 1, \alpha_2 = \alpha_2^*) = \pi_2^{NP}(\alpha_1 = 1, \alpha_2 = 1)$. *Then in the equilibrium of the quality-setting stage, firms employ the following strategies:*

1. *If* $\rho \geq \bar{\rho}$, *then firm 1 plays* $\alpha_1 = 1$ *and firm 2 plays* α_2^*, *which is a unique maximum of* $\pi_2^{NP}(\alpha_1 = 1, \alpha_2)$ *with firms playing price strategies according to Proposition 3.*
2. *If* $\rho \leq \bar{\rho}$, *then both firms play* $\alpha_i = 1$ *for* $i = 1, 2$.

The above characteristics provide unique equilibrium quality strategies for which at least one firm plays quality equal to 1.

According to Proposition 5, firms' equilibrium strategies depend both quantitatively and qualitatively on the scope of consumer inertia. If inertia is moderate, firms settle for an asymmetric equilibrium, in which one of the firms plays maximum quality while the other chooses inferior quality. However, if the value of ρ is low enough (lower than approximately 0.168), both firms decide to choose the maximum possible quality level. As mentioned, high inertia steers firms into improving their quality, as the benefits of attached consumers outweigh the downside of increased price competition. As shown in Figure 5, quality slightly decreases with ρ and is lowest if there is no inertia. In this case, $\alpha_2^* = 1/2$. Thus, higher inertia leads to overall higher product quality.

Figure 5. Inferior quality in equilibrium as a function of inertia parameter ρ



Source: Author's own elaboration.

Note that Proposition 5 defines the equilibrium strategy specifically for the case when $\alpha_1 = 1$. Thus, other equilibria may exist in which firms select quality levels strictly lower than one. However, it is important to note that the equilibrium in Proposition 5 exists uniquely across all values of the inertia parameter ρ. Additionally, it is constructively similar to the equilibrium found in the one-period models of Szech [2011] and Wisnicki [2022], making comparisons between these models straightforward.

Knowing the equilibrium strategies at every stage of the game, we can write down the equilibrium strategies' full description. Naturally, the form of the equilibrium strategies depends on the value of our only parameter of the game: the level of inertia. The following corollary summarises the equilibrium of the presented game.

Corollary 1. *Propositions 2, 3, 4, and 5 provide the following equilibrium strategies:*

1. *If* $\rho \geq \rho$, *firms play the following strategies:*
   - *Firms play quality* $\alpha_1 = 1$ *and* $\alpha_2 = \alpha^*$ *from point 1 of Proposition 5.*
   - *Inequalities (5) are satisfied. Firms play the first-period pricing strategies in the form of continuous distributions according to Proposition 3, with* $0 < J_1 < J_2$. *Firm 1 sets an atom at* $p = 1$.
   - *Firms play the second-period pricing strategies in the form of continuous distributions according to Proposition 2. Firm 1 is prominent and sets an atom at* $p = 1$. *No firm sets an atom at* $p = 0$.
2. *If* $\rho \leq \rho$, *firms play the following strategies:*
   - *Firms play quality* $\alpha_1 = 1$ *and* $\alpha_2 = 1$.
   - *Inequalities (5) are not satisfied. Firms play pure strategy* $p = 0$ *as the first-period pricing strategies. No firm is prominent* ($x_1 = x_2$).
   - *Firms play the second-period pricing strategies according to Proposition 2; that is, they play continuous CDF* F_i *of the form* $F_i(p) = \frac{\rho+1}{2\rho} - \frac{1-\rho}{2\rho} \cdot \frac{1}{p}$, *defined on the interval* $\left[\frac{1-\rho}{1+\rho}, 1\right]$.

In addition, each firm plays lower prices (in terms of FOSD) in the first rather than the second period.

With the addition of the quality selection stage, the equilibrium characteristics simplify. The piecewise strategy described in Proposition 4 no longer appears in the equilibrium profile. In addition, regardless of the type of quality equilibrium, firms play identical price strategies in the first period.

If inertia is modest, firms choose asymmetric qualities, and this choice implies their strategies in the game's pricing stages. As the first-period pricing strategies are identical, the firm that sets higher quality does not abuse its superiority through higher pricing in the first period. The firm then uses its advantage in the second period, when it sets a higher price compared to both its competitor and the previous period.

If consumer inertia is sufficiently high, firms adopt an extreme strategy by choosing the highest possible quality levels. This high quality, coupled with very high inertia, makes it highly unlikely for a consumer to switch between periods. Consequently, firms focus on obtaining a high market share in the first period and decide to offer their product for free at $t = 0$. They forgo profits in the first stage of the game, opting to split the market evenly[4]. This lack of first-period profit is offset in the second period, where firms employ mixed pricing strategies with prices strictly greater than zero. This pricing approach is a prime example of an "invest-harvest" strategy [**Farrell, Klemperer 2007**].

With the equilibrium strategies established, we can calculate firms' profits in equilibrium. These may be derived from the previous propositions; however, for convenience, we present the profit formulas to capture all stages of the game equilibrium.

Corollary 2. *In the equilibrium of the two-period game, firms obtain profits* π_1^*, π_2^* *of the following form:*

1. *If* $\rho \geq \bar{\rho}$:
$$\pi_1^* = \left(1-\alpha_2^*\right)\left(2-(1-\rho)\alpha_2^*\right),$$
$$\pi_2^* = \alpha_2^* \left(1 - \alpha_2^*\left(1+(1-\rho)\alpha_2\right) + \frac{\left(1-\alpha_2\right)\left((1-\rho)\alpha_2^* + \rho\right)\left(1+(1-\rho)\alpha_2^*\right)}{1-(1-\rho)\left(\alpha_2^*\right)^2} \right),$$
where $\alpha_2^* = \alpha_2^*(\rho)$ *is defined in Proposition 5.*
2. *If* $\rho \leq \bar{\rho}$: $\pi_1^* = \pi_2^* = \frac{1-\rho}{2}$.

Moreover, $\pi_1^* \geq \pi_2^*$, *and the inequality is strict for* $\rho < \bar{\rho}$.

[4] Note that, in equilibrium, there are no inactive consumers as at least one firm plays $\alpha = 1$.

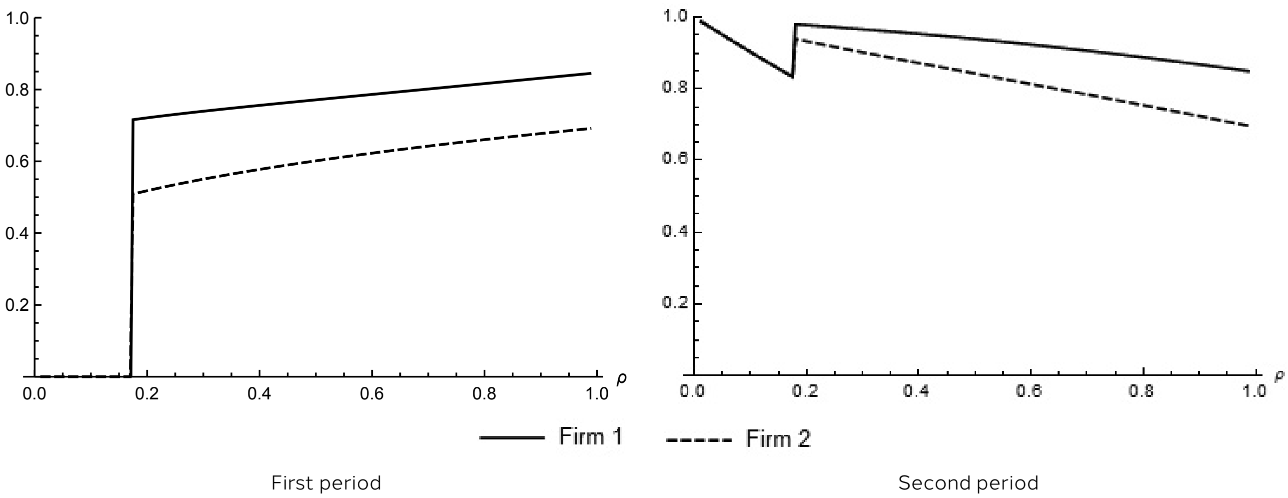
As intuition suggests, the firm with the higher quality set performs better profit-wise in equilibrium. In turn, we use the formulas from 2 in the subsequent section's comparative statics to determine the impact of inertia on the market.

Impact of inertia

The above analysis shows that inertia grants firms monopolistic power over attached consumers while potentially enhancing product quality. Thus, it may be essential to assess how inertia affects this type of market.

First, we investigate equilibrium market prices. Generally, inertia's effect on price distributions in equilibrium is not monotonic across the support of the strategies. Figure 6 shows the average prices set by firms at equilibrium. Inertia magnifies the "invest-harvest" effect: if consumers are deeply attached to a satisfactory product, firms try to lure them into buying their product in the first period and exploit their attachment in the subsequent period.

Figure 6. Average prices set by firms in the first (Figure A) and second (Figure B) period equilibrium as functions of inertia parameter



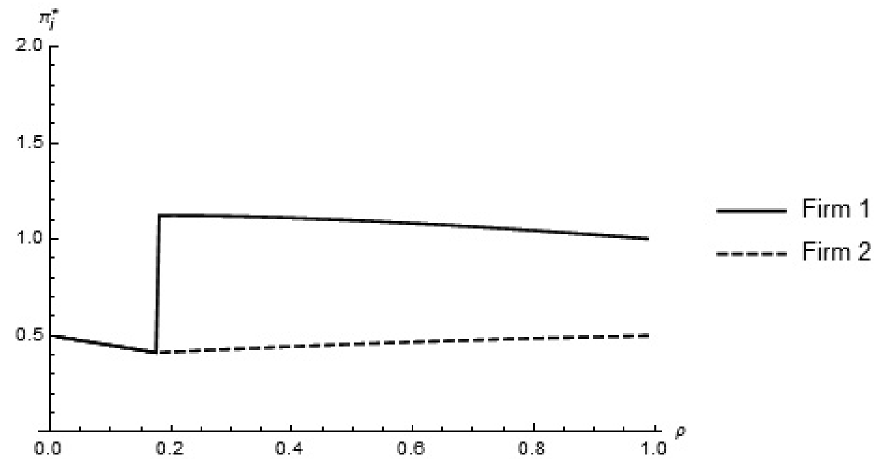
Source: Author's own elaboration.

The firm setting the higher quality also leverages its superior position by choosing a higher price than its opponent in both periods. This price difference between firms increases with inertia in the first period (when firms play varying quality levels) and decreases in the second period. As low inertia makes the quality gap between firms widen, the prominent firm may command a higher price in the second period after adopting a more competitive approach in the first. When inertia is very high, the equilibrium shifts to a symmetric configuration with an extreme "invest-harvest" strategy.

Next we investigate the effects of consumer inertia on market welfare. Specifically, we analyse how inertia impacts firms' profits in equilibrium. We utilise the profit formulas from Corollary 2.

Intuitively, inertia should increase the market share of the prominent firm at the expense of its competitor. Inertia makes the prominent firm's customers less likely to switch. However, the firm's prominence may come not only from market share but also from a considerable quality difference, as Proposition 1 suggests. Therefore, inertia might not necessarily lead to an increased market share in the second period if the prominent firm does not retain a sufficient number of attached consumers.

Figure 7. Firms' profits in equilibrium as functions of consumer inertia parameter ρ (assuming $\alpha_1 \geq \alpha_2$)



Source: Author's own elaboration.

Figure 7 depicts how much each firm earns cumulatively in equilibrium, conditional on the value of ρ. The effect of inertia on firms' profits is ambiguous both across firms and across different scopes of inertia. For the high-quality firm, the value of $\rho = \rho$ maximises its profits[5]. The firm also earns substantially less if consumer inertia is high. Thus, firm 1 gains from higher inertia as the firm can increase the second period's price and exploit the attached consumers. However, for $\rho < \overline{\rho}$ the extreme "invest-harvest" strategy of playing $p = 0$ in the first period is not very beneficial to firm 1. The firm has to forfeit first-period profits for a slightly higher return in the second period and ends up worse than when $\rho > \overline{\rho}$.

In contrast, for the firm offering slightly lower quality, the value of ρ at $\overline{\rho}$ represents its lowest profitability point. Unlike its competitor, firm 2 maximises profits when there is no inertia (so $\rho = 1$) or when inertia is at a maximum ($\rho \rightarrow 0$). If consumers do not exhibit any inertia, firm 2 plays a moderate price (identical in both periods) and gains some consumers in each period – even though it is inferior in quality – through lower prices. An increase in inertia forces firm 2 to increase quality and lower price in the first period. However, this price lowering does not compensate for the loss of market share due to higher attachment, as it benefits the firm offering the high-quality product. It is only when inertia is high enough that both firms choose maximum quality, and firm 2's profits increase with inertia. With no quality advantage, both firms can exploit consumer attachment by setting high prices in the second period.

In terms of total industry performance, these opposing effects of consumer inertia tend to balance out for $\rho > \overline{\rho}$. The maximum for firms' profits lies at approximately $\rho = 0.44$, or around the midpoint of the possible values that inertia can take. From an overall profit perspective, a much worse situation occurs when ρ is lower than $\overline{\rho}$. Then there is no extraordinary profit for the superior-quality firm, and the total profits are at their lowest. Thus, contrary to intuition, firms do not necessarily benefit from high inertia.

We now examine the impact of consumer inertia on total market welfare. In line with Spiegler [2006b] and Szech [2011], we describe welfare as the proportion of satisfied consumers in the market[6]. Since prices merely represent monetary transfers within the market, they are omitted in this calculation. Unlike previous studies, we must consider two periods in the analysis here. Thus, we define the total market as the undiscounted sum of the proportions of consumers who received a satisfactory product in both periods; accordingly, total welfare in the market, *TW*, takes values from 0 to 2.

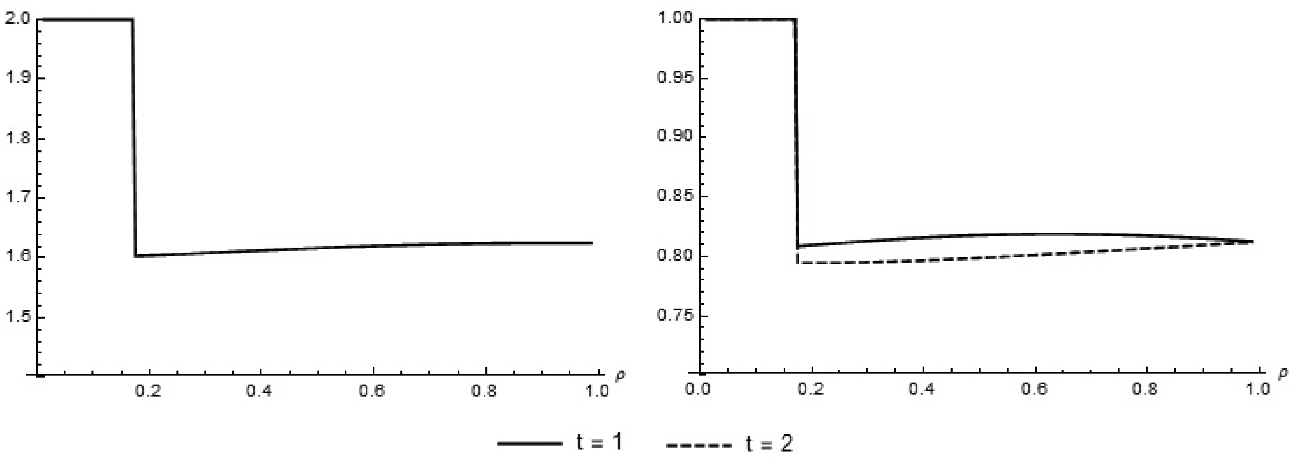
Note that all consumers are active in equilibrium, as at least one firm plays maximal quality. In other words, in equilibrium $x_0 = 0$. Hence, only two factors impact total welfare. First, the value of total welfare depends on the direct impact of product quality: as quality improves, the likelihood of satisfied consumers increases.

5 Provided that firms play the asymmetric equilibrium.
6 Note that the reservation prices (hence utility value in monetary terms) are normalised to 1.

Second, welfare is influenced by firms' market shares. Specifically, total welfare rises if more consumers purchase from the superior-quality firm. Thus, if both firms set quality at $\alpha_i = 1$, total welfare is maximised since all consumers are satisfied post-purchase in both periods. However, if inertia is low and we have the asymmetric equilibrium, inertia's impact on total welfare is not straightforward. The following corollary outlines the impact of inertia on total welfare:

Corollary 3. *For* $\rho < \overline{\rho}$, *total welfare decreases with the level of inertia (so it increases with* ρ *). The total welfare for* $\rho > \overline{\rho}$ *is maximal and equal to 2.*

Figure 8. Total welfare: cumulative in Figure (a) and divided into time periods in Figure (b) as functions of inertia parameter ρ



Cumulative | Divided into periods

Source: Author's own elaboration.

Figure 8 illustrates how total welfare changes with inertia, both overall and by time period. For $\rho > \overline{\rho}$, several factors shape the impact of inertia on total welfare. First, as inertia increases, so does α_2, leading to higher overall product quality in the market, which boosts total welfare. Conversely, the rise in quality for the second firm, combined with greater inertia, draws some consumers away from firm 1 (the higher-quality firm) towards firm 2, which provides a lower-quality product. This shift occurs primarily in the second period, where quality and consumer attachment both impact outcomes. As a result, the market share of firm 1 generally decreases as inertia increases. Specifically, in the first period, the market share initially rises and then starts to decrease as consumer inertia intensifies; in the second period, it steadily declines as ρ falls (as indirectly shown in Figure 7).

The product of these forces ultimately causes total welfare to decline with rising consumer inertia. The increase in quality for the firm selling the inferior product does not compensate for the shift in market shares, especially in the second period. However, the decrease in total welfare as a result of inertia is not very severe (see Figure 6) in comparison to the rise in maximum total welfare near $\overline{\rho}$. At this level, the quality of firm 2's product nearly doubles as inertia increases, indicating that very high consumer inertia generally boosts the market's total welfare.

Conclusion

This paper addresses the impact of consumer inertia on an oligopoly market. To capture inertia, we use a dynamic 2-period model in which consumers exhibit attachment to the offered products between periods. Moreover, consumers display limited comparability, which we use to facilitate inertia. We apply these behavioural consumer traits into a duopoly model of price competition with endogenous quality.

Our findings show that inertia has a profound effect on the market in dynamic equilibrium. Due to consumer attachment, firms adopt an "invest-harvest" pricing strategy: they lower prices in the first period to attract consumers, then exploit consumer attachment in the second period with high markups. Generally, increasing the scope of inertia magnifies the price difference as firms further leverage consumer attachment.

Apart from influencing price strategies, inertia also affects firms' quality decisions. Our equilibrium findings indicate that firms, on average, decide to increase product quality as inertia intensifies. This increase comes from the fact that quality plays a complementary role in consumer attachment; for consumers to exhibit inertia, they must be sufficiently satisfied with their current brand choice to dismiss alternatives. Consequently, firms opt for high quality if inertia is substantial. Moreover, the increase in quality itself may impact pricing by intensifying competition, leading firms to reduce prices.

A key aspect of this paper is examining the welfare implications of inertia. Although consumer inertia may intuitively be viewed as a flaw in rational and effective decision making, its market implications are more complex. Substantial inertia forces firms to improve quality and offer first-period price discounts. Consequently, consumers may benefit from developing substantial attachment to specific brands. By driving firms to enhance product quality, high inertia can maximise total market welfare.

While this paper sheds light on inertia's influence on consumer welfare, it has limitations. First, we analyse only a duopoly, which represents a specific type of oligopolistic market. A natural extension would be to generalise the analysis to an n -firm setting to explore the effects of competition on market welfare. Moreover, consumer behaviour could be modelled with greater heterogeneity. From a modelling perspective, a more comprehensive analysis of equilibrium qualities, including mixed strategies, would be valuable. Finally, a more in-depth dynamic model, such as one using Markov strategies, could be used to capture the dynamic nature of firms' decision making, without the end-game effect of a fixed-period model.

The insights from this model can be used to enhance market understanding and offer public policy perspectives. Often, public authorities, including antitrust regulators, seek to encourage consumers to make more active choices, aiming to drive competition among firms. However, this approach overlooks the quality aspect of inertia, as firms are more likely to invest in quality if they can secure a significant and stable consumer base. We hope this paper offers fresh perspectives on this debate.

References

Andersen S., Campbell J.Y., Meisner Nielsen K., Ramadorai T. [2015], *Inattention and Inertia in Household Finance: Evidence from the Danish Mortgage Market*, National Bureau of Economic Research.

Armstrong M., Vickers J. [2019], Discriminating Against Captive Customers, *American Economic Review: Insights*, 1 (3).

Bewley T.F. [1986], *Knightian Decision Theory*, vol. 1, Cowles Foundation for Research in Economics, Yale University.

Bewley T.F. et al. [1987], *Knightian Decision Theory*, vol. 2, Intertemporal Problems, Cowles Foundation.

Crawford G.S., Nicola Tosini N., Waehrer K. [2011], *The Impact of 'rollover' contracts on Switching Costs in the UK Voice Market: Evidence from Disaggregate Customer Billing Data*, The Warwick Economics Research Paper Series (TWERPS).

DellaVigna S. [2009], Psychology and Economics: Evidence from the Field, *Journal of Economic Literature*, 47 (2): 315–372.

Ericson K.M.M. [2014], *On the Interaction of Memory and Procrastination: Implications for Reminders*, National Bureau of Economic Research.

Ericson K.M.M., Fuster A. [2011], Expectations as Endowments: Evidence on Reference-Dependent Preferences from Exchange and Valuation Experiments, *The Quarterly Journal of Economics*, 126 (4): 1879–1907.

Farrell J., Klemperer P. [2007], Coordination and Lock-in: Competition with Switching Costs and Network Effects, *Handbook of Industrial Organization*, 3: 1967–2072.

Farrell J., Shapiro C. [1988], Dynamic Competition with Switching Costs, *The RAND Journal of Economics*, 123–137.

Goettler R.L., Clay K. [2011], Tariff Choice with Consumer Learning and Switching Costs, *Journal of Marketing Research*, 48 (4): 633–652.

Grubb M.D., Osborne M. [2015], Cellular Service Demand: Biased Beliefs, Learning, and Bill Shock, *American Economic Review*, 105 (1): 234–271.

Handel B. R. [2013], Adverse Selection and Inertia in Health Insurance Markets: When Nudging Hurts, *American Economic Review,* 103 (7): 2643–2682.

Kamenica E., Mullainathan S., Thaler R. [2011], Helping Consumers Know Themselves, *American Economic Review*, 101 (3): 417–422.

Masatlioglu Y., Ok E. A. [2005], Rational Choice with Status Quo Bias, *Journal of Economic Theory*, 121 (1): 1–29.

Masatlioglu Y., Ok E. A. [2014], A Canonical Model of Choice with Initial Endowments, *Review of Economic Studies*, 81 (2): 851–883.

Ortoleva P. [2010], Status Quo Bias, Multiple Priors and Uncertainty Aversion, *Games and Economic Behavior*, 69 (2): 411–424.

Osborne M. J., Rubinstein A. [1998], Games with Procedurally Rational Players, *American Economic Review*, 834–847.

Osterman P. [1987], Choice of Employment Systems in Internal Labor Markets, *Industrial Relations: A Journal of Economy and Society*, 26 (1): 46–67.

Sagi J. S. [2006], Anchored Preference Relations, *Journal of Economic Theory,* 130 (1): 283–295.

Samuelson W., Zeckhauser R. [1988], Status Quo Bias in Decision Making, *Journal of Risk and Uncertainty*, 1 (1): 7–59.

Shum M. [2004], Does Advertising Overcome Brand Loyalty? Evidence from the Breakfast-Cereals Market, *Journal of Economics & Management Strategy*, 13 (2): 241–272.

Simon H. A. [1955], A Behavioral Model of Rational Choice, *The Quarterly Journal of Economics*, 69 (1): 99–118.

Sitzia S., Zheng J., Zizzo D. J. [2015], Inattentive Consumers in Markets for Services, *Theory and Decision*, 79 (2): 307–332.

Spiegler R. [2006], The Market for Quacks, *The Review of Economic Studies*, 73 (4): 1113–1131.

Szech N. [2011], Becoming a Bad Doctor, *Journal of Economic Behavior & Organization*, 80 (1): 244–257.

Tripsas M., Gavetti G. [2000], Capabilities, Cognition, and Inertia: Evidence from Digital Imaging, *Strategic Management Journal*, 21 (10–11): 1147–1161.

Wisnicki B. [2022], Consumer Inertia Fosters Product Quality, *Journal of Behavioral and Experimental Economics*, 96: 101817.

Wisnicki B. [2023], *Choice Inertia in Industrial Organization with Boundedly Rational Consumers*, PhD thesis, SGH Warsaw School of Economics, College of Economics Analysis.

Appendix

Proof of propositions 1 and 2

Equilibrium follows the logic described in section 3.1. From **Armstrong and Vickers [2019]**, we know that the unique equilibrium must be in the form of continuous CDFs defined on the common interval $[L,1]$, where $L \in [0,1]$. Moreover, firms may set atoms at 0 or 1, but only at most one firm sets an atom at $p=1$. Hence, we must consider three possible cases: one in which firm 1 sets an atom at $p=1$, one in which only firm 2 does the same, and the third fringe case in which no firm sets an atom at $p=1$.

Assume that, if any firm sets an atom at $p=1$, it is firm 1. Then, firm 1 receives a sure payoff $\pi_{1,2}(1)$, as stated in Equation (3), if it sets price $p_{1,2}=1$ for any $p_2<1$. As $p=1$ belongs to the equilibrium support of firm 1's strategy, in equilibrium, firm 2 plays strategy $F_2(p)$ with the support $[L,1]$ such that firm 1 gets an identical profit from playing any pure strategy $p_1 \in [L,1]$. Therefore, the equation

$$\pi_{1,2}\left(p,F_2(p)\right)=\pi_{1,2}\left(p_{1,2}=1\right)$$

provides us with the equilibrium strategy of firm 2.

We obtain the lower bound of equilibrium strategy L by solving $F_2(p=L)=0$. If the calculated value is lower than zero, then the lower bound is zero. To find the equilibrium strategy of firm 1, observe that if firm 2 sets $p_{2,2}=L$, then it gains a sure profit of $\pi_{2,2}(L)$, as described in Equation (3), for any $p_1>L$ played by its opponent. Thus, in equilibrium, firm 1 needs to make firm 2's profit from playing any $p_{2,2}\in(L,1)$ identical to $\pi_{2,2}(L)$.

Hence, the formula of $F_1(p)$ is obtained by solving

$$\pi_{2,2}\left(p,F_1(p)\right)=\pi_{2,2}\left(p_{2,2}=L\right).$$

The atom size at $p=1$ is calculated from $1-F_1(1^-)$.

We perform the same procedure for reversing the firms, assuming that firm 2 plays an atom at $p=1$ if any firm does so. Let us denote by L_i the lower bound of the strategies' support, calculated by assuming that firm i sets an atom at $p=1$. If $L_1 < L_2$, then the atom that firm 2 would play as a result of this procedure is negative. Hence, it cannot be an equilibrium strategy, and vice versa. If $L_1 = L_2$, then no firm plays any atom and the two procedures coincide. If $\max\{L_1, L_2\} < 0$, then the strategies are defined on the unit interval and set atoms at $p=0$ of the size $F_i(0)$ for firm i, as negative prices are not valid.

The value of x^* in proposition 1 comes from equating $L_1 = L_2$. $x > x^*$ corresponds to $L_1 < L_2$.

Proof of proposition 3

By assumption, inequalities (5) are satisfied. Thus, the market share of firm 1 after the first period, x_1^P, is sufficiently high that even if firm 1 sets a higher price than firm 2 for every customer, it is still prominent.

As in the proofs of the previous propositions, we use **Armstrong and Vickers [2019]** and state that the equilibrium strategies are continuous distributions defined on a subset of the unit interval with the lower bound $\max\{J, 0\}$. We also consider two cases regarding the firm that can set an atom at $p=1$.

The proof follows the logic of the previous one. Assume that firm 1 is the only firm that can set an atom at $p=1$. If firm 1 plays $p_{1,1}=1$, then it gains a sure payoff $\pi_1(p_{1,1}=1)$ in the first period and the corresponding second-period payoff of a prominent firm. Firm 2 then plays a mixed strategy $G_2(p)$ defined on $\max J_2, 0$ such that it solves

$$\pi_1^P(p_{1,1}=1) = \pi_1^P(p, G_2(p)). \tag{6}$$

The value of J_2 comes by solving $F_2(p = J_2) = 0$.

The distribution $G_1(p)$ is obtained from solving

$$\pi_2^{NP}(p_{2,1} = J_2) = \pi_2^{NP}(p, G_1(p)), \tag{7}$$

as firm 2 is not prominent in the second period for any second-period price setting. The atom size at $p=1$ comes from calculating $1 - G_1(1^-)$. We can see that $\pi_1^P(p, G_2(p))$ and $\pi_2^{NP}(p, G_1(p))$ are continuous functions in $G_i(p)$ for $i = 1,2$. Moreover, they decrease in the value of $G_2(p)$ and increase in p. Hence, there are always values of $G_i(p)$ such that equations (6) and (7) are satisfied.

The same procedure can be applied assuming that firm 2 is the only firm that can set an atom at $p=1$. This results in obtaining the value of J_1 (analog of J_2) and also alternative formulas for $G_1(p)$, $G_2(p)$. If $J_2 < J_1$, then only the strategies in which firm 1 sets an atom at $p=1$ are valid; otherwise, the atom size would be negative. The reversed situation occurs if $J_2 > J_1$. If $J_1 = J_2$, then both methods coincide and no firm sets any atom at $p=1$.

If $\max\{J_1, J_2\} < 0$, then firms set atoms at $p=0$ of the size of $G_i(0)$ as the negative prices are not valid. The profit formulas come from $\pi_i(1)$ for firm i setting an atom at $p=1$ and $\pi_{-i}(\max J_i, J_{-i})$ for the other firm.

Proof of proposition 4

As inequality (5) is not satisfied, firm 1 is not prominent if it always plays a higher price than firm 2.

If $\pi_1^0 > \pi_1^{NP}(p_{1,1}=1, p_{2,1}=0)$, then $p=1$ cannot be in the equilibrium support of firm 1 since it is dominated by $p=0$. Moreover, $\pi_1^0 > \pi_1^N P$ implies $\pi_2^0 > \pi_2^{NP}(p_{2,1}=1, p_{1,1}=0)$, so 1 is not in equilibrium support of any firm. This means that the equilibrium strategy cannot have any $p>0$ in its support, as any $p \in (0,1)$ is dominated by $p=1$. Hence, the equilibrium strategy must be to play $p=0$.

Now, assume that $\pi_1^0 < \pi_1^{NP}(p_{1,1}=1, p_{2,1}=0)$. Then, the construction of the proof is similar to that of Proposition 3 with one exception. As prominence changes with the probabilities of setting a lower price than the opponent, we must consider two formulas that form the equilibrium strategies for different price ranges.

Assume that firm 1 is setting an atom at $p=1$ or that no firm sets it. The value of K_1 is obtained from

$$\pi^{NP}\left(p_{1,1}=1\right)=\pi^{P}\left(p_{1,1}=K\right).$$

Let us define G_2^* as the value of G_2 such that the profit of firm 1 is the same, regardless of prominence (equivalently, it is the probability of the price of firm 2 being lower than that of firm 1 for which $x_1 = x^*$). Then, we define p_1^* as the price solving

$$\pi_1\left(p_1^*,G_2^*\right)=\pi_1^{NP}\left(p_{1,1}=1\right),$$

which is the price at which prominence changes. In equilibrium, firm 2 needs to make the profit of firm 1 from playing any p_1 in the equilibrium support equal to $\pi_1^{NP}\left(p_{1,1}=1\right)$. Therefore, $G_2(p)$ comes from solving

$$\pi_1^{NP}\left(p,G_2(p)\right)=\pi_1^{NP}\left(p_{1,1}=1\right),\quad \text{for } p\geq p_1^*,$$
$$\pi_1^{P}\left(p,G_2(p)\right)=\pi_1^{NP}\left(p_{1,1}=1\right),\quad \text{for } p< p_1^*.$$

Notice that for $p=p_1^*$, $\pi_1^{NP}\left(p_1^*,G_2\left(p_1^*\right)\right)=\pi_1^{P}\left(p_1^*,G_2\left(p_1^*\right)\right)$.

We perform an analogous procedure to obtain the equilibrium strategy of firm 1. The difference here is that, to obtain the formula for $G_1(p)$, we equate π_2^k to $\pi_2^P\left(p_{2,1}=K_1\right)$. The existence of $G_1(p)$ can be deduced in the same manner as in the proof of Proposition 3.

Now, we assume that firm 2 is the one which can set an atom at $p=1$ and calculate the corresponding value K_2 and equilibrium distributions. If $K_1 < K_2$, then firm 1 sets an atom as the second procedure results in the atom of firm 2 being negative and vice versa. If $K_1 = K_2$, no firms set an atom at $p=1$ and the two methods coincide.

If $\pi_1^0=\pi_1^{NP}\left(p_{1,1}=1,p_{2,1}=0\right)$, then both playing $p=0$ and strategies in the form of the above-described distributions provide identical profits, hence both form equilibrium strategies.

Proof of proposition 5

First, let us assume that $\alpha_1 = 1$. Hence, all the possible cases of equilibrium described in Propositions 3 and 4 may occur for a given combination of α_2 and ρ, as shown in Figure 9.

We start with the case in which firm 1 is guaranteed prominence (Proposition 3) and it is the firm that sets an atom at $p=1$ in the second period. The assumption (5) becomes:

$$\alpha_2 \leq 1/2 \vee \frac{1}{1-2\alpha_2}+\frac{1}{\alpha_2}+1<\rho. \tag{8}$$

Let us assume now that the inequality (8) is satisfied and firm 1 also sets an atom at $p=1$ in the first period. For $\alpha_1 = 1$, this occurs if

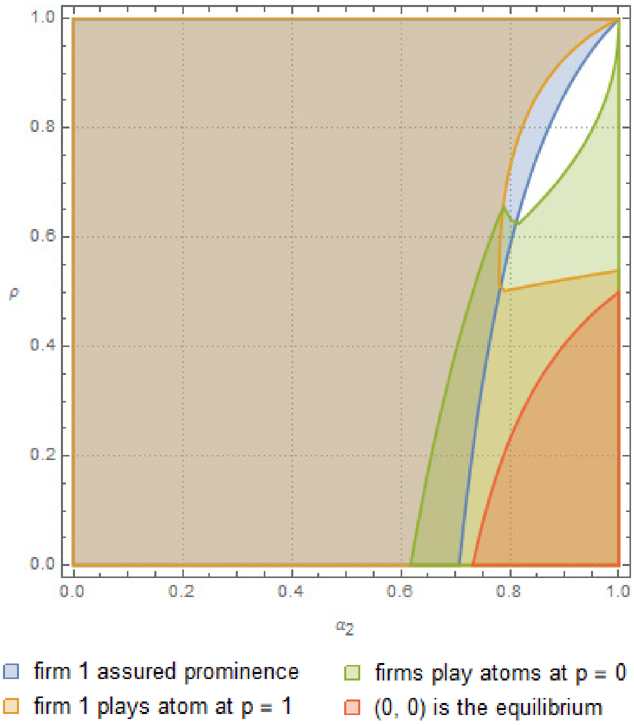
$$\left(\alpha_2 \leq 1/2 \vee \frac{1}{1-2\alpha_2}+\frac{1}{\alpha_2}+1<\rho\right)\text{and }\ \alpha_2^2\left(\alpha_2(\rho-1)^2(\alpha_2+\rho)+\rho^2+\rho-2\right)+1>\alpha_2\rho. \tag{9}$$

From direct calculations, it can be seen that, for any value of $\rho\in\left(0,1\right]$, there exists a $\alpha_2\in\left[0,1\right]$ such that inequality (9) is satisfied.

Firm 2 plays the pricing strategy in the first period according to Proposition 3 and obtains the following profit:

$$\pi_2=\alpha_2\left(\alpha_2\left(\alpha_2(\rho-1)-1\right)-\frac{\left(\alpha_2-1\right)\left(\alpha_2(\rho-1)-\rho\right)\left(\alpha_2(\rho-1)-1\right)}{\alpha_2^2(\rho-1)+1}+1\right).$$

Figure 9. Price equilibrium types by values of α_2 and ρ.



Source: Author's own elaboration.

In this formula, profit has a single optimum α_2^* inside the unit interval for a given value of ρ. Moreover, this optimum satisfies condition (9).

We denote by $\pi_2^*(\rho)$ the profit that firm 2 obtains if it plays α_2^*. We now compare π_2^* to optimal profits obtained from other equilibrium characteristics of Propositions 3 and 4.

If (8) is satisfied but firm 2 places an atom at $p=1$ in the second period, then the optimal level of α_2 is $\alpha_2 = \dfrac{1}{2\rho}$, which provides a constant profit of 1/4 for firm 2. This profit is always lower than π_2^* for any ρ in the unit interval.

Now, consider that (8) is not satisfied and we need to consider Proposition 4 as a price equilibrium. Specifically, let us focus on the case in which both firms set prices in the first period at zero. Then, the profit of firm 2, given that $\alpha_1 = 1$, increases in α_2. This means that the equilibrium candidate would be for both firms to set quality levels at 1. We denote the profit of firm 2 in this case as π_2^0.

The comparison between π_2^* and π_2^0 provides us with conditions that determine which of these strategies is the equilibrium one. The (1, 1) quality profile is the equilibrium strategy if $\rho < \rho$, where ρ is a unique value of $\rho \in [0,1]$ for which both profits are equal.

The condition that allows zero prices in the first period to be the pricing equilibrium, for $\alpha_1 = \alpha_2 = 1$ takes the following form:

$$2\rho^2 + 1 > 3\rho,$$

which is always satisfied for $\rho \le \bar{\rho}$.

Consider now that the conditions for the piecewise price equilibrium of Proposition 3 are met. Then, the profit of firm 2, regardless of whether it sets an atom at $p=1$, is always lower than π_2^* for any α_2 or ρ inside the unit interval.

Now, we revoke the assumption that $\alpha_1 = 1$. We can see that, if $\alpha_2 = \alpha_2^*$ and conditions for the equilibrium of Proposition 3 are met (with firm 1 setting an atom at $p=1$), then the profit of firm 1 increases with α_1 for any $\alpha_1 < 1$. In addition, if conditions for zero-price equilibrium are met, the profit of firm 1 also increases with α_1 if $\alpha_2 = 1$.