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Marcin Bielecki
Narodowy Bank Polski, Poland
(*corresponding author*)

Michał Brzoza-Brzezina
Narodowy Bank Polski, Poland

Aneta Błażejowska
Narodowy Bank Polski, Poland

Kamila Kuziemska-Pawlak
Narodowy Bank Polski, Poland

Grzegorz Szafrański
Narodowy Bank Polski, Poland

Estimates and Projections of the Natural Rate of Interest for Poland and the Euro Area

Szacunki i projekcje naturalnej stopy procentowej dla Polski i strefy euro

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Abstract

We use a wide range of models (time series, semi-structural and structural) to estimate, project and explain the evolution of the natural rate of interest (NRI) in Poland (PL) and in the euro area (EA). Our findings are as follows: (i) the NRIs declined significantly in both PL and the EA over the last two decades, (ii) our most recent short-term estimates point towards a mildly negative NRI in PL and appreciably more negative NRI in the EA, (iii) our most recent medium-term estimates point towards a mildly positive NRI in PL and a slightly negative NRI in the EA, (iv) the NRI in PL remained consistently above the NRI in the EA by an average of 2–3 pp, (v) the main drivers of the declining NRIs were demographics and the worldwide productivity slowdown, (vi) the main driver of the higher NRI in PL than in the EA was the productivity catch-up in PL, (vii) we expect the NRI in PL to significantly converge to the EA level by the mid-2030s.

Streszczenie

Wykorzystujemy szeroką gamę modeli (szeregów czasowych, półstrukturalnych i strukturalnych) do oszacowania poziomów naturalnej stopy procentowej (NSP) w Polsce i strefie euro oraz wskazania czynników odpowiedzialnych za ich zmiany w czasie. Nasze ustalenia są następujące: (i) NSP znacznie spadła zarówno w Polsce, jak i w strefie euro w ciągu ostatnich dwóch dekad, (ii) nasze oszacowania „krótkoterminowe" wskazują na umiarkowanie ujemną NSP w Polsce i znacznie bardziej ujemną NSP w strefie euro, (iii) nasze oszacowania „średnioterminowe" wskazują na nieznacznie dodatnią NSP w Polsce i nieznacznie ujemną NSP w strefie euro, (iv) NSP w Polsce utrzymywała się stale powyżej NSP w strefie euro o średnio 2–3 pkt proc., (v) głównymi czynnikami powodującymi spadek NSP były demografia i ogólnoświatowe spowolnienie produktywności, (vi) głównym czynnikiem wpływającym na wyższy poziom NSP w Polsce niż w strefie euro było nadrabianie zaległości w zakresie produktywności w Polsce, (vii) oczekujemy, że NSP w Polsce zrówna się z NSP w strefie euro do połowy lat 30. XXI w.

Słowa kluczowe:
Polska, strefa euro, polityka pieniężna, naturalna stopa procentowa

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Introduction

The natural rate of interest (NRI) is a central concept in modern monetary economics. Introduced more than a hundred years ago by Knut Wicksell, it has evolved and now constitutes one of the cornerstones of contemporary central banking. What is the NRI and why do we think that knowing its level is important? What are its strengths and weaknesses? What do we find in this paper? What did other economists find out about the NRI and how does our paper contribute to the field? These are the questions that we answer in this and the next sections.

The natural rate of interest

Wicksell [1898] defined the natural rate as the real interest rate that stabilises the price level. "There is a certain rate of interest on loans which is neutral in respect to commodity prices", he wrote, and "tends neither to raise nor to lower them. [...] It comes to much the same thing to describe it as the current value of the natural rate of interest on capital".

Simultaneously, he presented his view on the consequences of not adjusting the market interest rate to the natural rate. Wicksell wrote about a "cumulative process": should the market interest rate fall below the natural rate, investments would exceed savings. As a consequence, the economy would expand, prices would rise, and the real interest rate would fall, further stimulating the process.

Wicksell's concept remained on the peripheries of economics for the next 100 years. It returned to the mainstream in the 1990s, with the widespread introduction of inflation targeting by central banks. Under this monetary policy strategy, interest rates are the central banks' instruments, while inflation is the target. With a minor modification, the Wicksellian concept of the natural rate (redefined as the real interest rate that stabilises inflation) takes centre stage in modern central banking. If known, it allows the central bank to stabilise, raise or lower inflation by setting the policy rate at below or above the NRI respectively. It proves particularly useful after a period of disinflation, when policy rates need to be returned to a neutral stance in order to avoid deflation (if kept too high for too long) or a rebound of inflation (if lowered too much). As the central banks in Poland and the euro area combat inflation at the time of writing this paper, we hope that our estimates can play a helpful role in restoring a balanced macroeconomic path.

Like many other economic concepts, the NRI has strengths and weaknesses. Where do we see them? Let us start with the strengths. The concept organises thinking about monetary policy, and in particular about its stance. Paraphrasing Robert Lucas Jr., once you start thinking about monetary policy through the lens of the NRI, it is hard to think about anything else. Moreover, although the NRI is an unobservable variable, we have modelling techniques that allow for its estimation. We also have models that offer explanations for its evolution (see Section 3 for details).

What are the weak sides of the NRI? We see two. First, the variable is not directly observable and therefore subject to estimation errors. Our econometric estimates presented in Section 0 account for confidence bands and these can amount to more than 4 pp at the end of the sample. This is a lot and clearly reduces the applicability of the NRI in day-to-day policymaking.

Second, while so far we have referred to a particular definition of the NRI (the real interest rate that stabilises inflation), the practice is more nuanced (or even chaotic). Several approaches to defining, calculating and estimating the NRI co-exist in the literature. For instance, in structural models the NRI is usually defined as the real interest rate that would prevail under flexible prices and wages; e.g. **Woodford [2003]**. Other papers approach the NRI as the interest rate that keeps the output gap (itself another unobservable object without a unique definition) at zero and inflation stable; e.g. **Laubach and Williams [2003]**. Statistical approaches to estimating the NRI often equalise it with a trend (i.e. low-frequency component) in real interest rates and only loosely invoke economic theory; e.g. **Del Negro et al. [2017]**.

Instead of concentrating on one particular definition, we agnostically apply a wide range of economic and econometric models to estimate the NRI in Poland and in the euro area and check which features of the estimated NRI are robust across various approaches. The only distinction we exploit comes from labelling our estimates as short- and medium-run. The former rely mainly on financial market data (e.g. various interest rates) to estimate the NRI, while the latter use to a larger extent macroeconomic theory that links the NRI, e.g., to potential growth or demographic processes. We find this distinction useful to explain some features of the estimates and, in particular, differences between them.

We believe that our findings are interesting mainly if looked at jointly. Only then are we able to see the robust features of the NRI, but also understand where the differences come from. In what follows, we offer a bird's eye view of our main findings, by concentrating on the main common features of the estimated NRIs and explaining the differences encountered.

Overview of the results

We apply a wide range of models: time-series, semi-structural and structural models, to estimate the NRI in Poland and in the euro area.

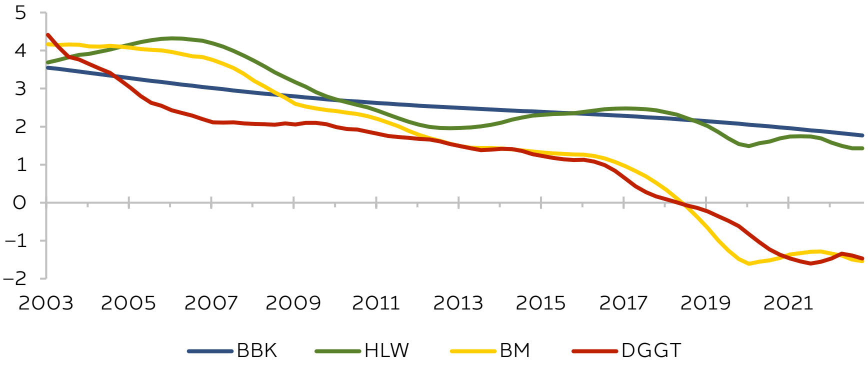
The most data-driven approach is based on decomposing time series into trend and stationary components, following the methodology of **Del Negro et al.** [**2017**], hereinafter referred to as DGGT. We select a representative open-economy model of the interest rates term structure, which will be presented in Section 3. The very nature of the decomposition is such that, in principle, the model ignores any structural macroeconomic relationships linking, for example, the NRI, the real interest rate and inflation but includes arbitrage-free relationships between financial yields in two economies. As a consequence, we argue that the estimated NRI likely reflects the financial markets' assessment of the natural rate.

Semi-structural approaches follow the models of **Holston, Laubach and Williams** [**2017**], hereinafter HLW, and **Brand and Mazelis** [**2019**], hereinafter BM. These models assume a backward-looking investment-savings (IS) curve and either accelerationist or partially forward-looking Phillips curves, as well as a relationship between the NRI and the potential growth rate of the economy. All relationships are loosely connected to standard micro-founded models, in particular the New Keynesian framework. The framework of **Brand and Mazelis** [**2019**] assumes additionally that the interest rate is driven by a Taylor-type monetary policy rule, which brings the model even closer to the New Keynesian framework. Importantly, this seemingly minor modification is far from innocent from a practical perspective. While the three relationships described before provide structural links between the NRI, the interest rate, output and inflation, the monetary policy rule adds the central banks' assessment on the likely level of the NRI to the econometricians' information set.

Our structural approach is based on a large quantitative life-cycle model of the euro area (hereinafter EA) and Poland (PL) in the vein of **Bielecki, Brzoza-Brzezina and Kolasa** [**2020**], hereinafter BBK. The model makes use of granular asset and demographic structures that are calibrated to reflect the main features of household balance sheets, fertility and longevity in both economies. Moreover, we take into account past and expected future developments in technology, which in particular reflect the rapid catching-up process that Poland underwent during the last 30 years, but also its expected slowdown in the future as the convergence process slowly matures. The NRI calculated using this framework is driven by demographic and technology processes, and in contrast to the previous approaches, neither reflects assessments by financial markets and central banks, nor is affected by business cycle developments.

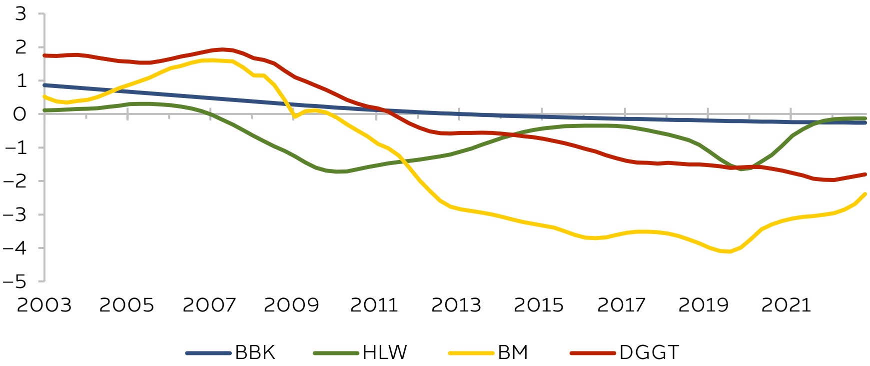
In what follows, we will briefly present our main findings in three dimensions. First, we show the estimates of the NRIs. Then, we discuss the divergence between the natural rates in Poland and in the euro area. Last but not least, we explain the main determinants which – in light of the structural model – underlie the trends of the NRIs. Details and more findings are presented in the next sections.

Figure 1. Estimates of the NRI in Poland (main results, quarterly data, %)



Source: Authors' own elaboration.

Figure 2. Estimates of the NRI in the euro area (main results, quarterly data, %)



Source: Authors' own elaboration.

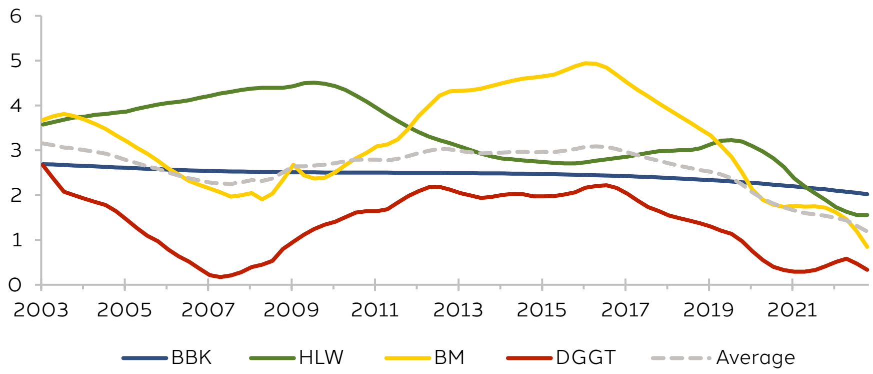
Figure 1 shows the selected estimates of the NRI in PL in 2003Q1–2022Q4 (the common part of our samples). Two findings stand out. First, irrespective of the model, a clear downward trend of the NRIs is visible. While in the beginning of our sample the natural rates vary between 3.5% and 4.4%, by the end of our sample these readings are much lower, between −1.5% and 1.8%. A similar trend is clearly visible for our estimates in the EA (Figure 2). Our findings are consistent with much of the literature discussed below, which documents a substantial decline of natural rates in both advanced and emerging market economies.

Second, the estimates of downward trends in the NRIs are clustered into two groups. The NRIs from the HLW and BBK models tend to move together, with the HLW measure hovering around the BBK, which stems from the medium-term fluctuations in the GDP trend growth rate. The NRIs from the remaining two models, i.e. BM and DGGT, trend downwards at a much faster pace, and diverge vividly from the first group. The DGGT measure consistently points toward comparatively lower levels of the NRI since 2005. This trend is further supported by the BM measure starting in 2013, coinciding with a period of persistently low inflation (and deflation) in Poland and the EA in the mid-2010s. The divergence between the two groups of estimates becomes statistically significant in 2015 and grows wider in the last part of our sample.

The BM and DGGT estimates take into account the assessment of the likely level of the NRI by the central bank and financial market participants, respectively, who additionally take into account the influence of economic forces that are largely outside of the information set of the NRI models based on medium- or long-run trends, i.e. HLW and BBK. These additional factors include the effects of the sovereign debt crisis for the weaker aggregate demand in the euro area, or the disturbances due to the COVID-19 pandemic and Russian aggression in Ukraine.

Our third conclusion results from comparing the NRIs in PL and the EA. We plot the differences between them in Figure 3. It is clear that, in spite of the substantial variability, the difference is one-sided. All models in all periods show a higher NRI in Poland. The average spread ranges from 1.9% to 3.6% and shows a slow downward trend throughout the sample period.

Figure 3. Difference between the NRI estimates in Poland and the euro area (main results, quarterly data, pp)



Source: Authors' own elaboration.

What explains our main findings, in particular the downward trends in the NRIs and the consistently higher NRI in Poland? Regarding the downward trend in the natural rates, our structural BBK model points to two main groups of factors. The first is demographics. Trends in the euro area and in Poland are similar: fertility rates are low and life expectancy increases. These factors contribute to higher per capita asset holdings and, as a consequence, lower the equilibrium real rate.

The second factor that contributes to the downward trend is the worldwide slowdown in productivity growth. Here, the trends in Poland and the euro area differ; nevertheless, as Poland is open to capital flows, the trend in the euro area contributes to lowering the country's NRI.

Differences in productivity play a key role in explaining the consistently higher NRI in Poland. Over the last 30 years, Poland has been catching up with the euro area. The country's GDP per capita increased from 38% of the euro area average in 1995 to 77% in 2022. The fast pace of productivity improvements, instrumental in this catch-up process, raised the equilibrium interest rate in Poland by approximately 2 pp above the euro area level.

Last but not least, beyond explaining the past, we intend to say something meaningful about the future. From the policymakers' point of view, what matters most is the current and future level of the NRI. We convey two messages. The first relates to the NRI in the near future, which might be of interest to policymakers as they attempt to combat inflationary pressures. Our range of estimates at the end of the sample is quite wide, and clearly it matters whether, for example, the NRI in Poland is +1.8% (as suggested by the BBK model) or within the –3.4% to 0.3% interval (as suggested by the 95% credibility intervals of the DGGT model). While it is difficult to judge unequivocally, it appears that the factors which recently caused the BM and DGGT model estimates below those of BBK and HLW have largely dissipated. New factors, such as higher fiscal expenditures and the potential AI revolution, which have not yet been fully incorporated into the information set of our models, suggest an upward trend[1].

The second message is about future developments. Forecasting the natural interest rates 10 or 20 years ahead is difficult but we believe that two relatively robust conclusions can be made. First, demographic forces (which are to a large extent predetermined over this horizon[2]) will continue to push the long-run trend of the NRIs downwards, both in the euro area and in Poland. Second, we should observe a gradual narrowing of the NRI gap between PL and the EA while the productivity gap slowly diminishes. Our projection is that the NRI gap will close by the mid-2030s to a significant extent. This has profound implications for Polish monetary

1 See also a recent debate between Blanchard and Summers in 2023.
2 Bielecki et al. [2022] argue that the impact of migration flows on the NRIs in the near term is limited.

policy. It suggests that there will be stronger downward pressure on the NRI in Poland, necessitating adjustments in monetary policy. Additionally, the probability of hitting the effective lower bound on interest rates should gradually increase in Poland.

Having discussed the main findings, we will now provide a brief overview of the relevant literature and outline our contribution.

Overview of the literature and our contribution

As already mentioned, the concept of the NRI rate came to prominence in the work of **Knut Wicksell** [**1898**], and it has been formalised in the context of modern macroeconomic theory by **Michael Woodford** [**2003**]. The natural interest rate, also referred to as the neutral or equilibrium interest rate, is an unobservable variable and several approaches have been developed to estimate it.

Among them, the one proposed by **Laubach and Williams** [**2003**] belongs to the most commonly used, cited and criticised[3]. The model builds on equations for the IS curve and the Phillips curve. It assumes that the NRI is driven by the trend growth rate of output and other factors, such as the households' rate of time preference. **Laubach and Williams** [**2003**] estimate jointly the NRI and potential output on US data using the Kalman filter and find that the NRI varied significantly from 1961 to 2002, with the variation in the trend growth rate being an important determinant of the NRI in the US economy. In a follow-up paper, **Holston et al.** [**2017**] estimate a version of the **Laubach-Williams** [**2003**] model using data for the United States, Canada, the euro area and the United Kingdom. They provide evidence of a downward trend in the NRI over 1990–2016. The substantial co-movement of country-by-country estimates over time suggests an important role of global factors in determining the NRIs.

Several papers have used the Laubach-Williams method and its modifications to estimate the NRIs for various countries. In particular, **Armelius et al.** [**2018**] find that the NRI in Sweden fell from 3% at the end of 1995 to –1.8% at the beginning of 2017. The Swedish NRI was influenced in both the short and long term by fluctuations in the US NRI. In turn, **Fujiwara et al.** [**2016**] show that the NRI in Japan began to decline in the 1990 s. It fell below zero during Japan's financial crisis of the late 1990s and during the global financial crisis in 2008. The rate hovered around 0% from 2010 to 2016Q1, after the introduction of "Quantitative and Qualitative Monetary Easing (QQE) with a Negative Interest Rate" by the Bank of Japan. **Arena et al.** [**2020**] find that the NRIs declined in virtually all European countries between 2000 and 2019, with some modest recovery after euro area debt crisis (2012). A significant part of that decline was accounted for by a fall in potential growth. Still, a wedge between the NRIs and the potential growth rates remained, which can be partly attributed to external factors. Finally, **Grigoli et al.** [**2023**], using the Laubach-Williams approach for 16 advanced countries over the period 1878 to 2019, find that natural interest rates exhibit three phases: i) from the 1870s to World War II they were relatively stable or slightly declining; ii) between WWII until the 1960s they were increasing; iii) since the 1960s they have declined steadily. The researchers document that for the median country, the decline since the 1960s' peak was 4.5 pp, bottoming out at 0.5% in 2019.

Another approach to estimate the NRIs is time-series decomposition. **Del Negro et al.** [**2017**] estimate a vector autoregression (VAR) with common trends for the United States between 1960s and 2010s. In line with the global saving glut hypothesis, they find a decline of the low-frequency component of the NRI in the United States since the late 1990s on account of a rising convenience yield for safe and liquid assets, and to a lesser extent, lower economic growth. In **Del Negro et al.** [**2019**], the analysis is extended to an open economy setup and applied to seven advanced economies dating back to 1870. The study documents that since the late 1970s the trend in the world real interest rate has essentially coincided with that of the United States. **Cesa-Bianchi**

[3] There are various criticisms of the use of the **Laubach and Williams** [**2003**] model. These range from issues with estimating the natural rate – such as the pile-up problem or filter uncertainty when either the IS or Phillips curve is flat; see **Fiorentini et al.** [**2018**] – to closely related identification problems; **Fiorentini et al.** [**2018**], **Bystrov** [**2019**].

et al. [2022] use the approach developed by Del Negro et al. [2019] to obtain estimates of the global equilibrium real interest rate in a panel of 31 countries from 1950 to 2015. In their simulation, the global equilibrium real interest rate rises from 1.25% in the mid-1950s to around 2.75% in the mid-1970s, declining steadily thereafter, reaching –0.25% in 2015. Michael Kiley [2020], in a paper on common global trends, attempts to compare and integrate term structure and small semi-structural models, while Johannsen and Mertens [2021] propose a flexible time-series model with macro variables and stochastic volatility to estimate shadow nominal interest rates when policy interest rates are below the zero lower bound.

Finally, the NRI can be calculated using a structural model. Krueger and Ludwig [2007] use a multi-country OLG model and predict a fall in the world interest rate due to demographic transition by approximately 1 pp between 2000 and 2060. Carvalho et al. [2016] calibrate a Blanchard-Yaari model to the average of several developed countries and simulate a more significant decline in the equilibrium interest rate in a shorter horizon (1.5 pp between 1990 and 2014). Bielecki et al. [2020] construct an open economy life-cycle model and investigate the impact of demographics on the NRI in the euro area. Demographic forces explain a decline in the NRI by about 2 pp between 1985 and 2030, of which about two-thirds can be attributed to population ageing. Eggertsson et al. [2019] use a 56-period life cycle model calibrated to match the US economy in 2015 and calculate the NRI ranging from –1.5% to –2.2%. The main drivers of the negative NRIs are population ageing, low fertility, and sluggish productivity growth. The main factor that has tended to counterbalance these forces is an increase in government debt. Finally, Platzer and Peruffo [2022] develop a heterogenous-agent overlapping generations model with non-homothetic preferences[4]. Their model can account for a 2.2 pp decline in the NRI in the United States between 1975 and 2015. The decline in the NRI is mostly due to the slowdown in total factor productivity growth, demographic factors, including a decrease in fertility and an increase in life expectancy, as well as rising income inequality. Growing public debt is, again, the major counteracting force.

Compared to the United States or the euro area, studies of the NRI for Poland are scarce. Brzoza-Brzezina [2006], using the Kalman filter and a structural VAR model, estimates the NRI for Poland between 1998 and 2003 and documents that the NRI averaged 4.6–5.0% over the analysed six years, showing relatively high variability. High productivity growth, high exchange rate risk and persistent budget deficits were the most probable causes of the high NRI in Poland at that time. Stefański [2018] investigates the NRI in three Central and Eastern European economies: Poland, the Czech Republic and Hungary, as well as in the euro area between 1996 and 2017. He uses an open economy New Keynesian model of Galí and Monacelli [2005] as a base to derive an extended, open economy version of the Laubach-Williams framework. Stefański [2018] documents that the NRI fell after the financial crisis, but rebounded afterwards, still remaining lower than in the pre-crisis peak (in Poland and the Czech Republic by 4–5 pp).

In turn, Acedański and Włodarczyk [2018] develop an open-economy version of the overlapping-generations model used by Carvalho et al. [2016], projecting a downward trend for the real interest rate in Poland due to ageing. They report that rapid ageing will reduce the interest rate gap between Poland and the developed economies by 1.3–2 pp. They also identify a strong positive relationship between interest rates and the retirement age and find that the decline in the interest rate will be caused mainly by an increase in savings related to the longer expected retirement duration. Bielecki et al. [2022] use an OLG model with nominal frictions to investigate how ageing and migration affect the Polish economy, in particular the NRI. They show that the decline in the NRI due to demographic processes is substantial, amounting to around 1.5 pp, albeit spread over a period of 40 years. The impact of migration flows is relatively small and cannot significantly alleviate the downward pressure on the NRI induced by population ageing.

To generalise and conclude, different studies, with respect to methods and countries covered, unequivocally find a decline in the NRIs over the past several decades. Estimates of the NRI tend to be imprecise and subject to real-time measurement error. Finally, changes in the NRI are driven by both macroeconomic factors –

4 A version of the model is also used in IMF [2023].

such as productivity growth, demographics, fiscal policy, and inequality – and financial factors, including international capital flows and the supply and demand for safe and liquid assets.

How does our paper relate to existing studies, and where do we see our contribution? While the models we use are not novel and have been applied before, they have typically been developed for closed economies. Our paper extends these models consistently to reflect the open economy feature of Poland.

In the time-series model setup, as in **Del Negro et al. [2017]**, we consistently estimate the NRI in Poland and the euro area using a variety of interest rates, including home and foreign interest rates, policy rates, and short- and long-term government bonds. Following **Del Negro et al. [2019]**, we use an assumption of uncovered interest rate parity to compare investment in euro-denominated Treasury securities in Poland and the euro area. We are able to decompose the changes in the nominal interest rate into the NRI, the cyclical part of the interest rate and permanent component (i.e. trend) of inflation on a quarterly basis. At the same time, we provide valuable information on fluctuations in the convenience yield on government bonds, and time premia.

In both the time-series and semi-structural approaches to NRI measurement, we use the most up-to-date estimates of policy interest rates for periods of quantitative easing[5]. We follow the current research trend of incorporating shadow rates into the analysis when the nominal interest rates are at the zero lower bound (ZLB). Additionally, we propose a tractable and relatively "prior-free" method of dealing with the unusual sequence of shocks affecting economies starting from early 2020. These shocks make estimating the standard HLW-like models quite tricky at the end of the current sample (see the description of the semi-structural models in Section 3).

In our structural life-cycle model, we treat Poland as a small open economy interacting with the large closed economy of the euro area. A novel feature of this paper is allowing both economies to systematically differ in their productivity levels, while the small open economy gradually catches up and slowly closes the historical productivity gap. This catch-up process turns out to be crucial in explaining the systematic differences between the NRIs in Poland and the euro area over our sample period.

Existing studies usually apply a single method to estimate the NRI. In contrast, we use a wide spectrum of methods, thus allowing to draw conclusions that are robust to such a test. Last but not least, as evidenced above, the NRI has been only scarcely covered for Poland. Our study provides policymakers in Poland and the euro area with up-to-date, robust knowledge about one of the most important objects in monetary economics.

Models

In this section, we present the main features of our modelling frameworks: the time-series models, the semi-structural models, and the structural model. A more detailed description of the models and the data used can be found in the Appendix.

Time-series models

In the time-series approach, we use non-structural models to disentangle common trends from a number of financial market time series. Hence, we decompose each of n quarterly times series, collected in row vector y_t, into their permanent, $\bar{y}_t$, and transitory components, $\tilde{y}_t$. As in the model of **Del Negro et al. [2017]**, each of the latent variables is generated from either a multivariate random walk or a stationary VAR and has a state space representation:

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t, \quad \text{measurement equation} \tag{1}$$

$$\bar{y}_t = \bar{y}_{t-1} + e_t, \quad \text{transition equation for trend components} \tag{2}$$

5 Our results, however, are quite robust to the use of the alternative approach with the model-consistent "shadow rate," i.e. by applying missing observations for the ZLB period in Poland – see Appendix.

$$\tilde{y}_t = \Phi^{-1}(L)\varepsilon_t, \quad \text{transition equation for transitory components} \tag{3}$$

where y_t is an $n \times 1$ vector of observables, both $\bar{y}_t$ and $\tilde{y}_t$ are row vectors of latent variables with q and n elements respectively, Λ is a $n \times q$ selection matrix with all elements restricted to be zero or one, and $\Phi(L) = I - \sum_{l=1}^{p} \Phi_l L^l$ is an inversible lag polynomial of order p. In this framework, $q \leq n$, the rank of Λ determines the number of common trends in observable variables, while $n - q$ is the number of cointegrating relationships in the system. The $q + n$ shocks are identically normally distributed:

$$\begin{bmatrix} e_t \\ \varepsilon_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0_q \\ 0_n \end{bmatrix}, \begin{bmatrix} \Sigma_e & 0 \\ 0 & \Sigma_\varepsilon \end{bmatrix} \right), \quad \text{NIID shocks} \tag{4}$$

where $N(0, \cdot)$ denotes the multivariate Gaussian distribution, with zero mean and positive definite covariance matrices Σ_e and Σ_ε.

The model (1–4) is an independent trend-cycle decomposition which resembles the multivariate decomposition of **Stock and Watson [1988]** with shocks to trend being orthogonal to shocks to transitory components. In the model, we also account for missing observations in y_t, and initial values in $\bar{y}_t$ and $\tilde{y}_t$. For the brevity of exposition, we omit technical details of Bayesian inference and postpone information on hyperparameters to the Appendix (Table 2). We start with a closed-economy model and then enhance it to an open-economy setup.

Closed-economy model

A model with inflation, policy-, short- and long-term interest rates in a closed economy setup for the Polish economy consists of the following measurement equations:

$$\pi_t = \bar{\pi}_t + \tilde{\pi}_t, \tag{5}$$

$$\pi_t^e = \bar{\pi}_t + \tilde{\pi}_t^e, \tag{6}$$

$$R_t = \bar{r}_t + \bar{\pi}_t + \tilde{R}_t, \tag{7}$$

$$NBP_t = \bar{\pi}_t + \bar{r}_t - \overline{cy}_t + \widetilde{NBP}_t, \tag{8}$$

$$I_t = \bar{\pi}_t + \bar{r}_t - \overline{cy}_t + \overline{tp}_t + \tilde{I}_t, \tag{9}$$

where π_t denotes headline inflation (CPI), π_t^e inflation expectations or (alternative version) the inflation target, R_t the short-term interbank interest rate (WIBOR), except for the ZLB period when it is substituted with the shadow rate estimates based on the **Krippner [2013]** method (or with missing values in the first alternative version of the model); NBP_t is the policy interest rate (with missing values during the ZLB period), and I_t is the 5-year Treasury bond interest rate.

We allow for divergence between the risk-free (Treasury or policy) and risk-bearing (interbank) interest rates. The difference, labelled convenience yield[6], $\overline{cy}_t$, lowers the interest rate of government bonds compared to the interbank interest rates of similar characteristics. The trend component of the real interest rate, $\bar{r}_t$, common to all interest rates is our measure of the NRI and one of the three stochastic trends that govern the dynamics of the model in the long run, next to the inflation trend, $\bar{\pi}_t$, and the term premium for longer maturities trend, $\overline{tp}_t$. Cyclical components, $\tilde{y}_t$, describe the short-run deviations of observed variables from their respective trends.

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[6] Our interpretation of the convenience yield is not exactly in line with **Del Negro et al. [2017]**. Still, the convenience of purchasing government bonds versus investment in the interbank market has its origin in safety and liquidity features as in **Del Negro et al. [2017]**.

Open-economy model

To enhance the model with open-economy features, we take into account two sources of spillovers from the EA, to the interest rates and to the exchange rate. We utilise the no-arbitrage (uncovered interest rate parity) condition, equation (13) from **Del Negro et al. [2019]**, to introduce an equilibrium relationship between the yield on Poland's government bonds, I_t^{EUR}, and euro-area government bonds, I_t^{EA}, both denominated in euros:

$$I_t^{EUR} = \left(\overline{r}_t + \overline{\pi}_t\right) + \overline{tp}_t - \overline{cy}_t^{EUR} + \overline{cy}_t - \overline{x}_t + \tilde{I}_t^{EUR}, \tag{10}$$

In the equation above, $\overline{cy}_t^{EUR}$ is the convenience yield on bonds denominated in EUR, and $\overline{x}_t$ is the permanent component of the return on the nominal bilateral exchange rate (PLNEUR).

Hence, equation (10) states that the long-term interest rate on Polish bonds denominated in the euro increases when either the domestic natural real interest rate, trend inflation, or term premium[7] increases, the convenience yield on euro-denominated bonds decreases or the exchange rate appreciates.

Symmetrically, we assume a common trend in headline inflation (HICP) and the inflation target in the EA analogously to equations (5) and (6), and we model the interest rate for the euro area in a similar vein as in equations (8) and (9):

$$ECB_t = \overline{r}_t^{EA} + \overline{\pi}_t^{EA} - \overline{cy}_t^{EUR} + \widetilde{ECB}_t, \tag{11}$$

$$I_t^{EA} = \overline{r}_t^{EA} + \overline{\pi}_t^{EA} + \overline{tp}_t - \overline{cy}_t^{EUR} + \tilde{I}_t^{EA}, \tag{12}$$

where ECB_t is a an approximation of the ECB policy interest rate obtained with the Krippner (2013) method, I_t^{EA} is the long-term interest rate on EA bonds, and $\overline{r}_t^{EA}$ and $\overline{\pi}_t^{EA}$ are trends of the interest rate and inflation in the EA respectively.

The second source of external shocks are exchange rate fluctuations. We decompose the annual dynamics of the exchange rate into a permanent and transitory component:

$$x_t = \overline{x}_t + \tilde{x}_t, \tag{13}$$

where $\overline{x}_t = \overline{x}_{t-1} + e_t^x$.

In the second alternative version of the model (FEER), we additionally relate the trend in the exchange rate, $\overline{x}_t$, to the fundamental equilibrium exchange rate, FEER, calculated externally, in the spirit of **Kuziemska-Pawlak and Mućk [2022]**:

$$FEER_t = \overline{x}_t + \widetilde{FEER}_t. \tag{14}$$

To this end, we keep the prior variance of $\widetilde{FEER}_t$ close to zero to shrink a common exchange rate trend, $\overline{x}_t$, in (13) and (14) towards the dynamics of FEER.

We estimate the open-economy DGGT models with a lag order in VAR, $p = 4$. The details on the priors and the initial values for latent variables can be found in the Appendix (Table 2).

Semi-structural models

The vantage point of the semi-structural models is the baseline New Keynesian framework with sticky prices. The non-policy equations, log-linearised around a steady state with the inflation target $\overline{\pi}$, can be expressed in the following manner:

$$\tilde{y}_t = \tilde{y}_{t+1}^e - a_r\left(i_t - \pi_{t+1}^e - \overline{r}_t\right) + \epsilon_t^{\tilde{y}}, \quad \text{New Keynesian IS curve} \tag{15}$$

7 For a matter of parsimony, we also assume that the respective time premia in financial markets for government bonds denominated in EUR and PLN are close to each other.

$$\pi_t = b_1 \pi_{t+1}^e + (1-b_1)\overline{\pi} + b_y \tilde{y}_t + \epsilon_t^{\pi}, \quad \text{New Keynesian Phillips curve} \tag{16}$$

where, in this model, $\tilde{y}_t$ and i_t denote the output gap and the nominal interest rate respectively. According to equation (15), the current output gap is a function of its future expected value, the difference between the ex-ante expected real interest rate and the NRI, as well as a demand-type shock $\epsilon_t^{\tilde{y}}$. In equation (16), current inflation is a function of inflation expectations, the central bank's target, the current output gap, and cost-push-type shock ϵ_t^{π}. The parameters a_r, b_1 and b_y can in principle be derived from fundamentals such as household preferences, production function parameters or the degree of price stickiness, see e.g. **Galí [2015]**.

The NRI of the baseline New Keynesian model is defined by:[8]

$$\overline{r}_t = \rho + \Delta \overline{y}_{t+1}^e, \quad \text{New Keynesian NRI} \tag{17}$$

where ρ is households' discount factor and $\Delta \overline{y}_{t+1}^e$ is the expected rate of growth of flexible price output.

The popular semi-structural approach of Laubach and Williams (later extended in HLW) recasts the above forward-looking system into its backward-looking analogue. Flexible price output is replaced by the GDP trend, and in both the IS and Phillips curve relationships, the expected terms are replaced with the (averages of) past values of the variables in question.

The block of GDP trend $\overline{y}_t$, output gap and the IS curve in the HLW model can be expressed using our notation as:

$$y_t = \overline{y}_t + \tilde{y}_t, \quad \text{GDP trend-cycle decomposition} \tag{18}$$

$$\overline{y}_t = \overline{y}_{t-1} + g_{t-1} + \epsilon_t^{\overline{y}}, \quad \text{GDP trend evolution} \tag{19}$$

$$g_t = g_{t-1} + \epsilon_t^g, \quad \text{GDP trend growth rate evolution} \tag{20}$$

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \tilde{y}_{t-2} - \frac{a_r}{2}\left(r_{t-1} - \overline{r}_{t-1} + r_{t-2} - \overline{r}_{t-2}\right) + \epsilon_t^{\tilde{y}}, \quad \text{HLW IS curve} \tag{21}$$

where the growth rate of GDP trend g_t follows a random walk, the GDP trend itself is also potentially affected by random level shifts $\epsilon_t^{\overline{y}}$, while the current output gap is a function of its past two realisations and past two real interest rate gaps.

Inflation in the HLW model follows an accelerationist version of the Phillips curve, and is a function of its lagged values, past output gap and a cost-push shock:

$$\pi_t = b_1 \pi_{t-1} + \frac{(1-b_1)}{3}\left(\pi_{t-2} + \pi_{t-3} + \pi_{t-4}\right) + b_y \tilde{y}_{t-1} + \epsilon_t^{\pi}. \quad \text{HLW Phillips curve} \tag{22}$$

The real interest rate used in equation (21) is constructed by subtracting the average inflation rate over the course of the past year from the current nominal interest rate:

$$r_t = i_t - \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right). \quad \text{HLW real interest rate} \tag{23}$$

The natural real interest rate is obtained as a sum of the GDP trend growth rate g_t and a catch-all component z_t, reflecting, for example, shifts in ρ, assumed to follow a random walk as well:

$$\overline{r}_t = g_t + z_t, \quad \text{HLW NRI} \tag{24}$$

$$z_t = z_{t-1} + \epsilon_t^z. \quad \text{Evolution of } z \tag{25}$$

8 For simplicity of exposition and following common practice, it is assumed that the elasticity of intertemporal substitution equals 1.

One shortcoming of the HLW model is its assumption of the nominal interest rate being a fully random variable. As a result, identification of the NRI level is typically quite weak, with large error bands, as the model relies ultimately on information obtained from the IS curve, where the slope parameter is subject to the pile-up problem (see e.g. **Fiorentini et al.** [**2018**]). Therefore, while the HLW procedure can recover the medium-run level of the NRI, it cannot be sufficiently informative about the movements in the NRI at high frequency.

To counteract this deficiency, **Brand and Mazelis** [**2019**] propose a setting where the nominal interest rate level results from conscious decisions made by an inflation-targeting central bank. The HLW framework is amended as follows. First, the Phillips curve relationship assumes that inflation is anchored at the inflation target:

$$\pi_t = (1-b_1)\bar{\pi} + \frac{b_1}{2}(\pi_{t-1} + \pi_{t-2}) + b_y \tilde{y}_{t-1} + \epsilon_t^{\pi}. \quad \text{BM Phillips curve} \tag{26}$$

Second, the central bank follows a Taylor-type rule when setting the nominal interest rate:

$$i_t = \rho_1 i_{t-1} + (1-\rho_1)\left(\overline{r}_t + \bar{\pi} + \rho_\pi(\pi_t - \bar{\pi}) + \rho_y \tilde{y}_t\right) + \epsilon_t^i. \quad \text{BM Taylor rule} \tag{27}$$

Importantly, this policy rule specification assumes that the average nominal interest rate level carries valuable information regarding the evolution of the NRI. An inflation-targeting central bank, after, for example, setting interest rates too low relative to the NRI, observes above-target inflation and is able to correct the course by hiking the policy interest rate. As a result, the identification of the NRI strongly relies on central bank actions, and this is the primary source of the difference in BM results compared to the HLW.

Since now inflation expectations can be defined within the model, the real interest rate entering the IS curve is the nominal interest rate less model-consistent expected inflation:

$$r_t = i_t - E_t \pi_{t+1}. \quad \text{BM real interest rate} \tag{28}$$

Finally, we tackle the issue of the difficulties of trend-cycle decomposition of GDP in periods of unusual and large shocks. **Holston et al.** [**2023**] propose a COVID-adjusted version of their model, utilising additional information on lockdown restrictions and allowing for enhanced shock volatility in 2020–2022. A drawback of their approach lies in fine-tuning a parameter which translates the lockdown stringency index to shifts in pandemic-adjusted potential GDP. As a result, their NRI estimates were not published during the pandemic period and until 2023.

Instead, we employ additional time-series information to aid the trend-cycle decomposition in real time, which we view as a more universal approach. Specifically, we rely on survey data on capacity utilisation in manufacturing[9]. As the survey data reported for a given quarter is gathered before the quarter ends, we assume that a part of the survey response refers to the previous period. We extend the HLW and BM models with the following additional equation linking output gap with the cyclical component of capacity utilisation[10]:

$$cu_t = c_0 \tilde{y}_t + c_1 \tilde{y}_{t-1} + \epsilon_t^{cu}. \quad \text{Capacity utilisation information} \tag{29}$$

[9] We would like to thank an anonymous referee for that suggestion. **Brand and Mazelis** [**2019**] use data on unemployment rates for a similar purpose. Due to various stimulus policies in 2020–2021, unemployment rates do not provide reliable signals on the output gap during the pandemic. The previous working paper version of the article (**Bielecki et al.** [**2023**]) used the **Hamilton** [**2017**] filter instead.

[10] While detrending of capacity utilisation data is not necessary in the case of the euro area, the series for Poland exhibits strong trends. We apply the Hodrick-Prescott filter with a smoothing parameter of 1600 to obtain the cyclical component of capacity utilisation.

Structural model

According to standard neoclassical single-sector growth models, the NRI depends predominantly on household preferences, as well as on variables exogenous from the point of view of the model, such as the rate of technological progress and the population growth rate. For example, in a variant of the Ramsey model, the long-run NRI can be expressed as[11]:

$$\overline{r} = \rho + n + x, \quad \text{Long-run NRI} \tag{30}$$

where x is the rate of technological progress. While it is not possible to characterise the NRI in a single, closed-form equation when households are heterogeneous, the same intuition regarding the NRI drivers still applies. In our BBK framework, the household sector is modelled as an aggregate of 80 cohorts, aged from 20 to 99, that are subject to time- and age-dependent mortality risk. As their lives are divided into periods of labour market activity and retirement, worker-consumers, in order to smooth their consumption over time, accumulate assets until they become retired, and later draw them down to support their old-age consumption. When life expectancy increases over time, it lowers the effective discount rate ρ and leads households to accumulate more assets.

Other important drivers of the observed (and projected) drop in the NRI encompass changes in pension system generosity that exacerbate the life-cycle savings motive due to declines in the pension to labour income ratio, low fertility resulting in declining or negative adult population growth rate n, and, last but not least, a slowdown in the rate of technological progress x.

The full description of the BBK model can be found in **Bielecki et al. [2020]**, and the complete set of model equations is reproduced in the Appendix. Compared to the original BBK model, we introduce two modifications to better reflect the historical experience of Poland.

First, while the TFP level of the EA follows productivity improvements at the world technology frontier, the Polish economy initially operates under a relatively low level of TFP. In line with historical data, we assume that since the 1990s Poland has experienced a rapid catch-up. Second, to capture the effects of the gradual opening of the Polish economy to international trade and financial market integration before EU accession, we assume a time-varying country risk-premium coefficient.

Results

Time-series models

We use Bayesian methods to approximate the low-frequency component of the NRI by a common trend in interest rates, $\overline{r}_t$, from the DGGT model. The time-series models use quarterly financial data[12] from Poland and the EA for the period 2001Q1–2022Q4, which brings the financial markets perspective on the NRI. The general outcomes are relatively robust to reasonable changes in the model specification (see two alternative versions in the Appendix).

Statistical inference in the baseline version of the DGGT model indicates that there is a downward trend in the NRI in Poland over the last 20 years (Figure 4). The median estimate of $\overline{r}_t$ decreased by 5.9 pp from 4.4% in 2003Q1 to –1.5% in 2022Q4. The 95% highest posterior density (HPD) interval of the change in $\overline{r}_t$

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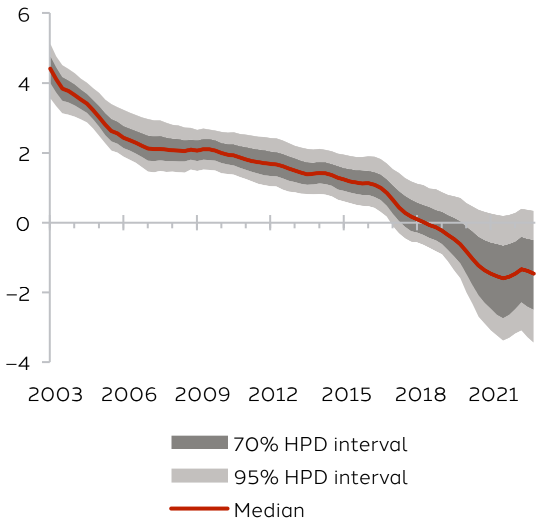
11 This equation results from the households' Euler equation when the welfare of subsequent cohorts is not weighed by their population (a so-called "average utilitarian" perspective). A similar but less elegant expression can also be obtained in the standard two-period OLG model, where $\overline{r} = \left[\alpha/(1-\alpha)\right](2+\rho)(1+n)(1+x)-1$, which can be approximated as $\overline{r} \approx \rho/2 + n + x$ for $\alpha \approx 1/3$.

12 In the baseline version of the DGGT model, these are: 3-month WIBOR (complemented with the shadow interest rate), NBP policy rate (with the 6-quarter ZLB period treated as missing data), interest rates on 5-year government bonds, annual inflation (CPI), and its NBP target, and their equivalents for the euro area (the shadow policy EBC rate, 5-year euro area government bonds, HICP inflation and its ECB target), additionally interest rates on 5-year Polish Treasury bonds denominated in euros, and the bilateral exchange rate of the EUR/PLN.

in the analysed period ranges from –8.2 pp to –3.7 pp. However, it is not certain whether the NRI from the DGGT model was negative in any of the last 12 quarters as the zero line stays within the 95% HPD interval. The 95% HPD intervals for the NRI level are twice as wide at the end of the sample (above 3 pp) than in the middle, which, to a certain extent, is a feature of the estimation method (filtering and smoothing step in the Gibbs sampler).

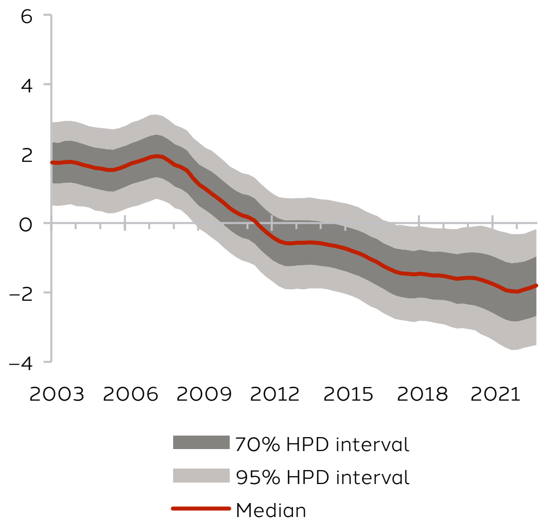
At the same time, the results confirm the downward NRI trend in the EA (Figure 5). Between 2003 and 2022 the NRI corresponding to the real ECB policy rate decreased by 3.6 pp. The 95% HPD interval of this change ranges from –4.9 pp to –2.2 pp, and it is highly probable that the NRI in the EA has been negative, at least since the first quarter of 2017.

Figure 4. Estimates of the NRI in Poland, baseline DGGT model (quarterly data, %)



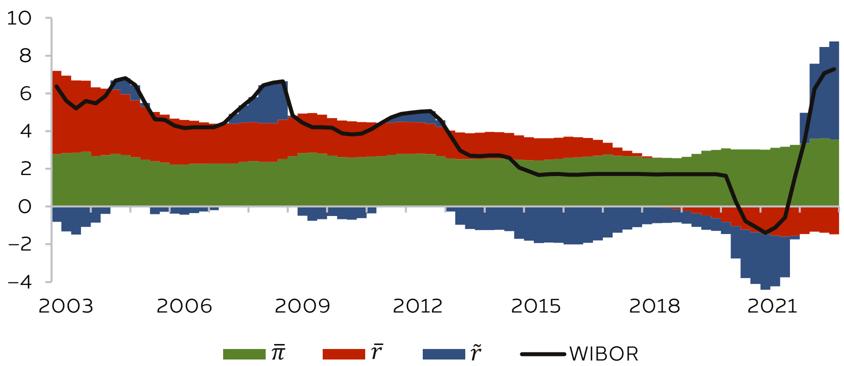
Source: Authors' own elaboration.

Figure 5. Estimates of the NRI in the euro area, baseline DGGT model (quarterly data, %)



Source: Authors' own elaboration.

Figure 6. Decomposition of WIBOR, baseline DGGT model (quarterly data, pp)

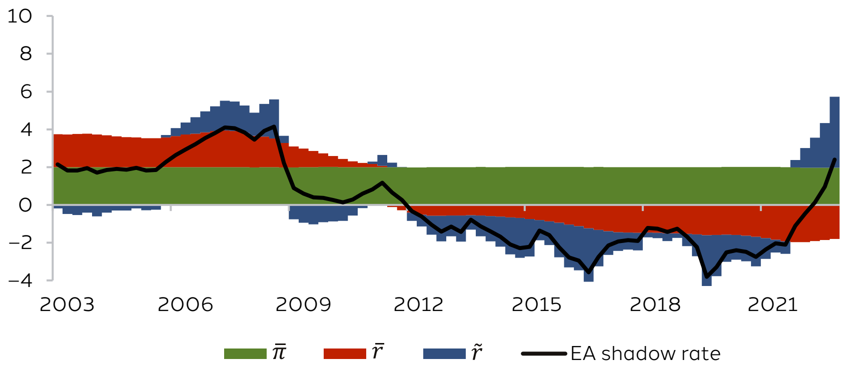


Source: Authors' own elaboration.

Time-series models do not allow for an economic interpretation of the NRI movements. Nevertheless, we see two areas where some conclusions can be drawn from looking at the estimated latent trend components (Figure 34 in the Appendix) and interest rate decomposition (Figure 6 and Figure 7). The first observation is that, in contrast to the NRIs, these components are relatively stable, both in PL and in the EA. The biggest fluctuations can be observed for the inflation trend in PL, which (not surprisingly) declines during the final disinflation period 2001–2002, then stabilises close to the NBP inflation target (2.5%) and rises in the final four years of our sample. In contrast, the inflation component in the EA is remarkably stable at approximately

2% throughout the sample. Both the term premium and the convenience yields for Polish bonds decline somewhat in the run-up to the Global Financial Crisis and increase afterwards.

Figure 7. Decomposition of the shadow interest rate in the euro area, baseline DGGT model (quarterly data, pp)



Source: Authors' own elaboration.

Figure 6 decomposes the Polish interbank rate (WIBOR) into the NRI, trend inflation and cyclical components. This exercise makes it possible to draw further conclusions. First, the bulk of the decline of WIBOR between 2003 and 2021 can be attributed to the financial markets' perception of the declining equilibrium real interest rate. Second, the recent sharply increasing interest rate is primarily perceived as a cyclical monetary policy tightening. In contrast, between 2013 and 2021, the interest rates were recognised as being below the equilibrium level (i.e. expansionary). Third, in the last part of our sample, the inflation trend is above the NBP inflation target (3.6% in 2022Q4). This means that the nominal equilibrium rate from the DGGT model (approximately 1.8% in 2022Q4) is actually higher than a simple correction of the real NRI for the inflation target would suggest.

Figure 7 shows an equivalent decomposition for the EA shadow interest rate[13]. Two things stand out. First, the medium-term downward forces that drive the trend of the nominal rates are steadily pushing down the NRIs in both Poland and the euro area. Second, the assesment of policy stance is roughly similar to that for Poland. In particular, monetary policy was perceived as expansionary between 2012 and 2021 and became contractionary in 2022. Last but not least, it should be noted that, likewise for the NRIs, there is also a systematic divergence of the nominal equilibrium rates, with the one in PL being substantially above that in the EA. The most recent estimate of the nominal equilibrium rate in the EA hovers around zero. This points to a higher probability of the EA facing the effective lower bound problem once the current inflationary episode is over.

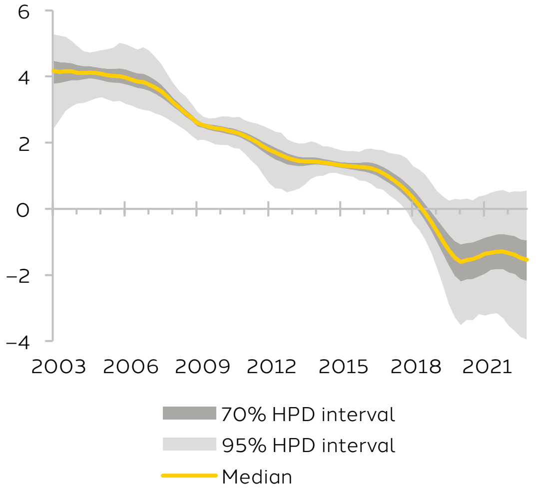
Semi-structural models

Both semi-structural models are estimated using Bayesian methods on the 1995Q1–2022Q4 sample for Poland and the euro area. The estimation procedure uses data on real GDP, core inflation and the interbank nominal interest rates[14]. Due to the unusual sequence of shocks experienced by both economies starting in 2020, we additionally supply the models with information on the cyclical component of capacity utilisation throughout the entire sample period (see Appendix for details on the Bayesian procedure and priors).

13 For brevity of exposition, we omit the contribution of the euro convenience yield, which is mostly stable throughout the sample and stays at the level of 1.5 pp.

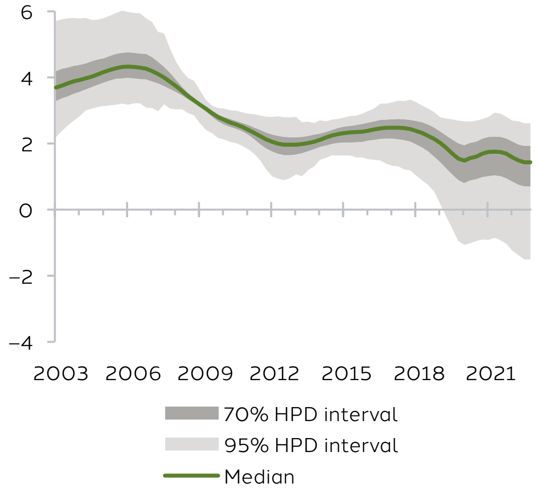
14 The baseline model uses Krippner [2013] and Hertel et al. [2022] shadow rates during the ZLB episodes. An alternative estimation, discussed in the Appendix, treats the ZLB period interest rates as missing data. The main results are robust to this change.

Figure 8. Estimates of the NRI in PL, BM model (quarterly data, %)



Source: Authors' own elaboration.

Figure 9. Estimates of the NRI in PL, HLW model (quarterly data, %)

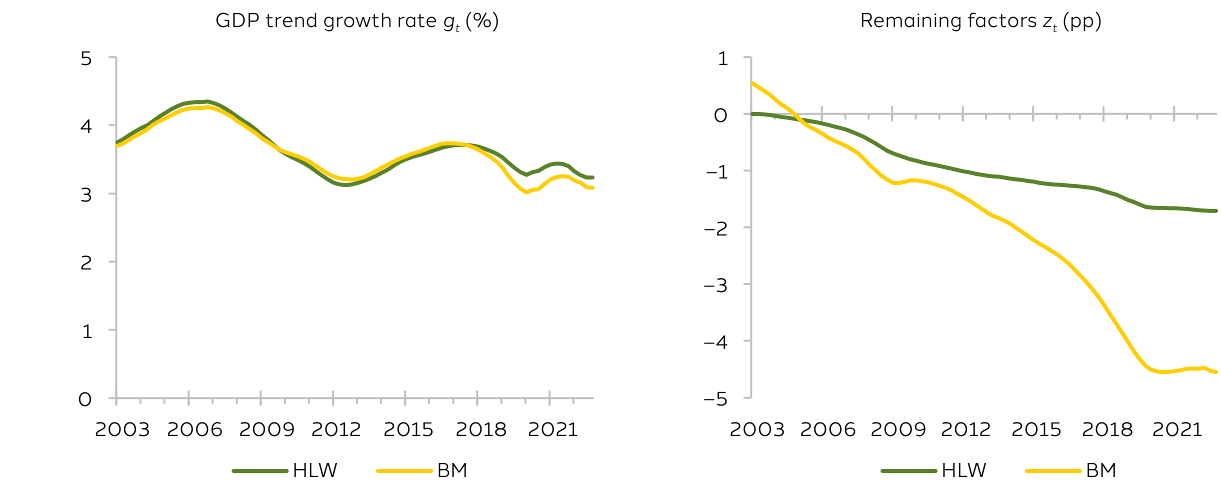


Source: Authors' own elaboration.

Similar to the DGGT, the BM model (Figure 8) finds a downward trend in the Polish NRI, declining from the median estimated value of 4.2% in 2003Q1 to –1.5% in 2022Q4, hence an overall drop of around 5.7 pp. Admittedly, the 95% HPD intervals are wide, especially at the beginning and end of our sample. A big part of the uncertainty in the NRI level is the consequence of the estimated Taylor rule parameter uncertainty.

In comparison, the HLW model (Figure 9) suggests a modest decline in the NRI, which also occurs at a non-uniform pace, reflecting medium-frequency fluctuations in the underlying growth rate of the GDP trend. The median estimate decreases from 3.7% in 2003Q1 to 1.4% in 2022Q4, constituting an overall drop of around 2.3 pp. Given these contrasting outcomes of related frameworks, a discussion of the sources of the discrepancy is in order. As seen in the Appendix, the estimated parameters common to both models are largely in line, and the extracted processes of output gap or inflation expectations are also alike. The two models also recover an almost identical GDP trend growth rate. The two approaches markedly differ in their interpretation of the random-walk catch-all component z_t. While the HLW model generates a gradual decline of around 1.7 pp since 2003Q1, in the case of the BM model the drop is much more pronounced, reaching around 5.1 pp (Figure 10).

Figure 10. Estimates of the drivers of the NRI in Poland, BM and HLW models (quarterly data)



Source: Authors' own elaboration.

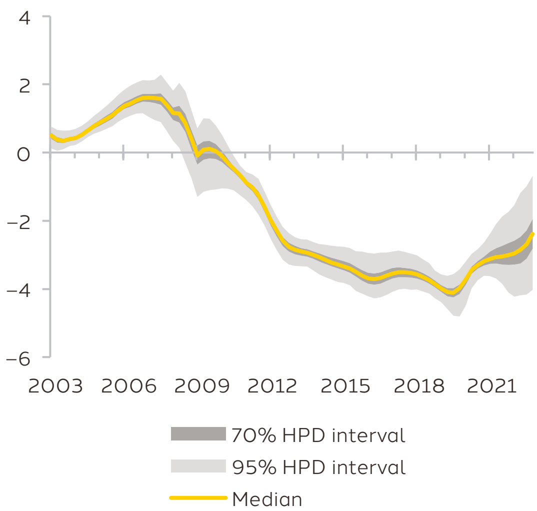
Source: Authors' own elaboration.

What accounts for this discrepancy is that the BM model, by taking into account the policy rule, adds the policymakers perspective to our information set. This may carry valuable (though most probably short-term) information about the natural level of both nominal and real interest rates.

The HLW model lacks the ability to reliably pick up the NRI short-run movements and therefore is more suitable for determining the medium-run evolution of the NRI. Both the DGGT and BM models can additionally uncover the NRI short-run movements (assessed by financial market participants and the central bank), although these movements can, in principle, also arise due to the misinterpretation of the "true" processes or the influence of additional shocks.

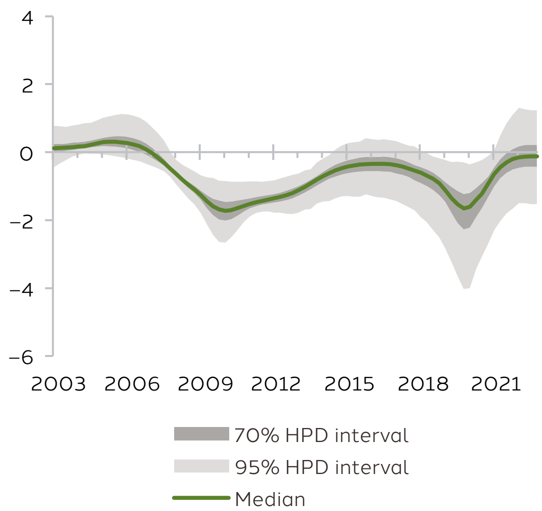
In line with the above interpretation, the BM method, when supplied with the shadow rate estimate of the EA money market conditions, finds a pronounced fall in the NRI after the financial crisis and over the entire decade of the 2010 s, which was first plagued by the sovereign debt crisis, and later suffered from aggregate demand deficiency. As a result, the obtained NRI estimate typically moves below the real interest rate implied by the shadow rate over the 2010s period and reaches its low point of less than –4% at the onset of the pandemic. Notably, the BM estimate picks up shortly afterwards and starts converging toward the DGGT and HLW estimates. The HLW estimate again follows quite closely the fluctuations in the growth rate of the GDP trend, dropping around the financial crisis, recovering in the second half of the 2010 s decade, falling again during the pandemic, and rebounding quite strongly in 2021 and 2022.

Figure 11. Estimates of the NRI in the euro area, BM model (quarterly data, %)



Source: Authors' own elaboration.

Figure 12. Estimates of the NRI in the euro area, HLW model (quarterly data, %)

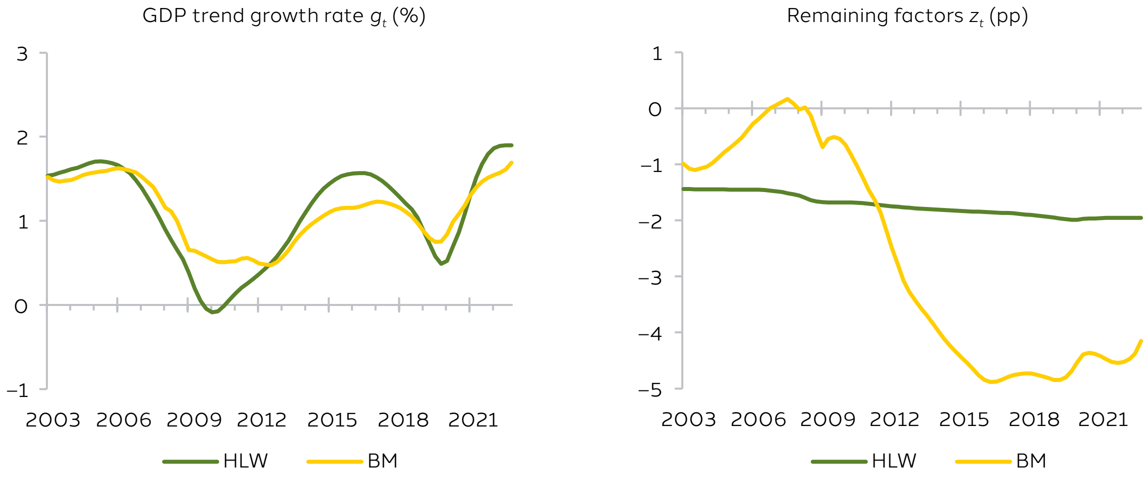


Source: Authors' own elaboration.

Over the last 20 years, the BM estimate of the median NRI for the euro area declined from 0.5% in 2003Q1 to –2.4% in 2022Q4, with an overall drop of around 2.9 pp (Figure 11). The HLW method finds a modest overall decline from 0.1% in 2003Q1 to –0.1% in 2022Q4 (Figure 12). The fall is much more pronounced if we take as a starting period 1999Q1, with the median NRI estimate of 1.4%.

Again, while the HLW and BM approaches do not differ substantially regarding the GDP trend growth rates, they recover staggeringly different levels of the catch-all z_t process (Figure 13). According to the BM model, the latter is subject to a dynamic decline since the onset of the financial crisis, becoming even more negative during the sovereign bond crisis in the EA, and only starting to stabilise in 2016 and recover from 2021 onwards. Such a sequence of events aligns quite well with our interpretation of the short-run NRI movements driven by various shocks whose influence dissipates over the medium run.

Figure 13. Estimates of the proximate drivers of the NRI in the euro area, BM and HLW models (quarterly data)



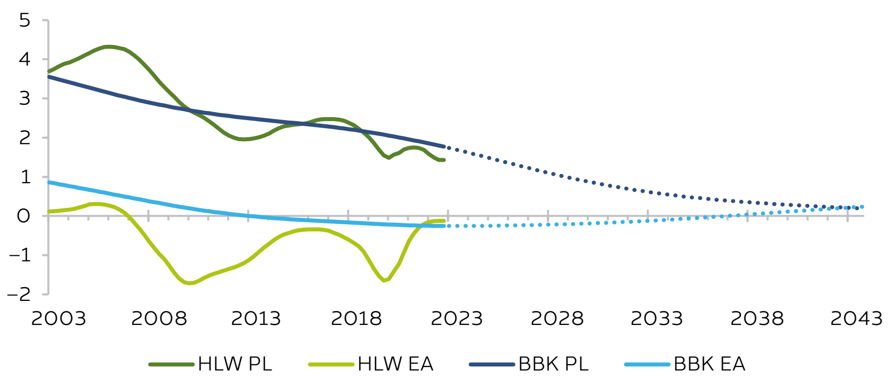
Source: Authors' own elaboration.

Structural model

While the semi-structural models can provide a proximate statistical decomposition of the movements in the NRI, their reliance on the random-walk assumption governing the underlying processes precludes a deeper economic interpretation of the obtained results. To interpret the observed decline in the NRI via the lens of measurable economic processes, we make use of the structural overlapping generations framework. Another advantage of this approach is to provide some intuition about the likely evolution of the NRI in the foreseeable future, as some trends that drive the data (especially demographic ones) are slow-moving or even largely pre-determined.

We first present the comparison of the "in-sample" estimates of the NRI obtained via the HLW method and from the BBK model (Figure 14). The HLW estimates fluctuate over the medium-run around a longer-run BBK trend, supporting our short-, medium- and long-run interpretation of the obtained results. The obtained estimates of the NRI from the BBK model for Poland suggest a decline from 3.6% in 2003Q1 to 1.8% in 2022Q4, an overall drop of around 1.8 pp. The decline in the euro area is more modest, from 0.9% in 2003Q1 to –0.3% in 2022Q4, an overall drop of more than 1 pp.

Figure 14. Estimates of the NRI in Poland and in the euro area, BBK and HLW models (quarterly data, %)

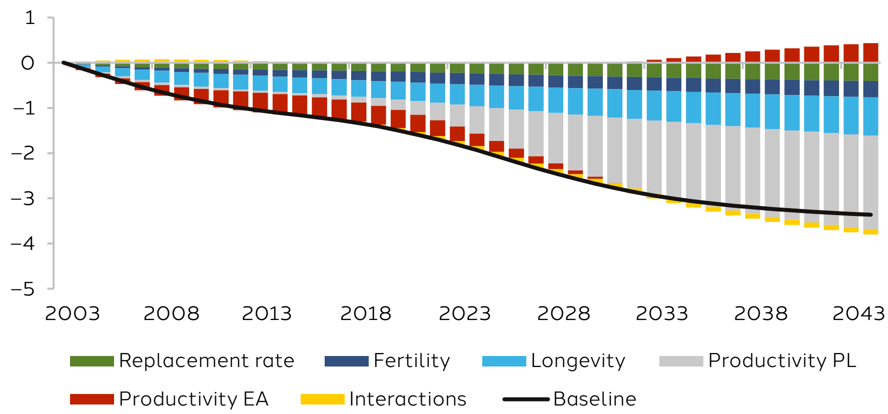


Source: Authors' own elaboration.

A unique advantage of the structural model is its ability to provide a glimpse into future developments. First, according to our projections, the NRI in Poland will probably continue its decline, stabilising slightly above 0% in the late 2030s. On the other hand, we do not expect a further decline in the EA NRI over the 2020–2040 period largely due to the expected pick-up in the rate of technological progress compared to the 2010s. As a consequence, we forecast a gradual convergence of the NRI in PL towards current EA levels, with most of the process completed by the mid-2030 s. While we cannot claim to encompass all possible factors driving the evolution of the NRI, our model provides a rich framework that includes several salient forces established in the literature as drivers of the long-run decline in the NRI in both advanced and emerging economies. The full life-cycle household sector allows us to account for changes in both fertility and mortality rates. Additionally, incorporating the pension system enables us to observe how households react to expected changes in the replacement rate (the ratio of the first pension benefit to the last-received labour income). Moreover, the inclusion of the TFP catch-up process and the open-economy environment enables us to incorporate the effects of the expected decline in the pace of productivity growth as the Polish economy converges to the EA, as well as important spillovers from the forces governing the evolution of the NRIs in advanced countries.

The bulk of the observed decline in the NRI over the 2003–2022 period can be attributed to two groups of forces (Figure 15). The first group is demographic in nature. It reflects the gradual "greying" of society due to declines in fertility, increases in longevity and, in consequence, declines in the expected pension system replacement rates. All of these factors induce households to accumulate more assets towards their retirement period, shifting the asset supply schedule rightwards and reducing the equilibrium real interest rate. The second group consists of declines in the growth rate of productivity, mainly due to the substantial EA productivity slowdown in the decade of the 2010s[15].

Figure 15. Structural decomposition of the decline in the NRI in Poland, BBK model (quarterly data, pp)



Source: Authors' own elaboration.

For the next two decades, we expect that the strongest driver of the decline in the Polish NRI will be the maturing of the convergence process, which will reduce the GDP trend growth rate in Poland to the growth rates of EA economies. This will reduce the relative attractiveness of investing in physical capital in Poland, resulting in a leftward shift of the asset demand schedule, and a lower NRI.

Finally, our projections envision a gradual pick-up in the pace of productivity improvements in the EA compared to the 2010s decade. These developments may become even more likely if the artificial intelligence revolution indeed takes place. However, they should not markedly influence the NRI gap between Poland and the EA, unless there are significant differences in the pace at which new technological improvements are adopted.

[15] For a decomposition of the declines in the EA NRI see Bielecki et al. [2020].

Conclusions

The natural rate of interest is a central concept for central banking as it determines the boundary between contractionary and expansionary monetary policy. Its importance increases in periods of extraordinary developments, such as the surge in inflation observed worldwide since 2021. In this paper, we use a wide range of models to provide a comprehensive view of the natural interest rates in Poland and the euro area. The applied models range from statistical time-series decompositions based on financial market data to semi-structural models based on macroeconomic data, and a structural life-cycle model based on demographic and productivity data. Our main findings are as follows:

- The NRIs declined significantly in both Poland and the euro area over the last 20 years.
- The main drivers of the declining NRIs were demographics and a productivity slowdown in the euro area. Low fertility rates and rising longevity in both the euro area and Poland contributed to higher per capita asset holdings and lower equilibrium interest rates. The productivity slowdown in the euro area affected the NRI in Poland due to openness to capital flows.
- We consider our estimates based on the structural model and one of the semi-structural models as the most likely indicators of where the NRI will be over the medium run. These models point towards a mildly positive NRI in Poland and a slightly negative NRI in the euro area.
- The estimates of the time-series model and our second semi-structural model that rely on additional information carried by short- and long-term nominal interest rates enable us to assess the likely level of the short-run NRI, with estimates pointing to mildly negative NRI in Poland and appreciably more negative in the euro area. As both economies entered a period of strong aggregate demand after the outbreak of the COVID-19 pandemic, we believe that the disturbances that were keeping the short-run NRI estimates comparatively low have largely dissipated. We expect that the short-run measures will gradually converge upward towards their medium-run counterparts as more data points from the post-pandemic period become available.
- The NRI in Poland remained consistently above the NRI in the euro area by an average of 2–3 pp throughout our entire sample period. This was mainly driven by the productivity catch-up in Poland. As the productivity gap significantly narrowed by 2022, we expect a relative productivity slowdown in Poland in the future and, as a consequence, gradual convergence of the NRI levels. Conditional on this assumption, the NRI in Poland is expected to have converged to the euro area level to a large degree by the mid-2030s.
- We expect demographic forces to exert a downward pressure on the NRIs in both Poland and the euro area in the foreseeable future. Regarding productivity growth, the picture is less clear. Recent advances in the usage of artificial intelligence may have the potential to speed up productivity growth significantly. If this effect materialises, technology growth may start to raise the NRI which, as a result, may increase in the euro area. This effect, however, seems unlikely to outweigh the expected productivity slowdown in Poland, so the NRI is more likely to decline further in the future.

Overall, our paper delivers a robust set of findings about the NRI in Poland and the euro area, which we hope will aid in the conduct of monetary policy.

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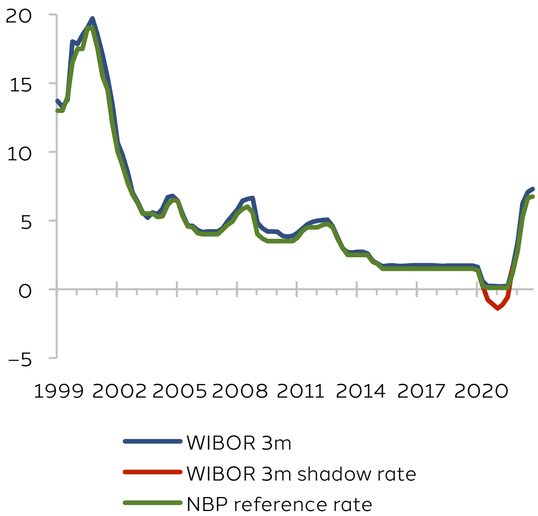
Appendix

Data

Time-series models

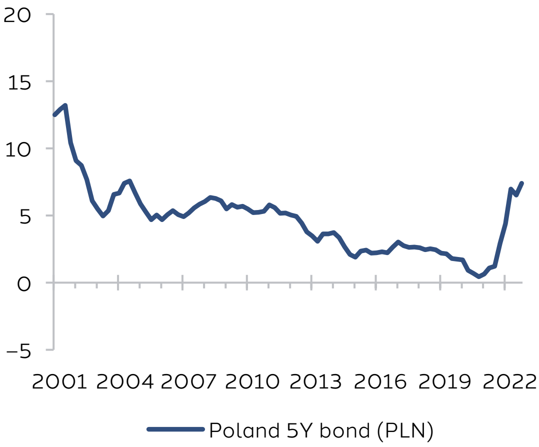
The dataset of financial and macroeconomic variables covers the period from 2001Q1 to 2022Q4, with the exception of Polish government bonds denominated in EUR (since 2013Q2). The data are taken from Bloomberg (by default), official statistics or research data sources (if explicitly mentioned) and recalculated from daily frequency to quarterly averages. The variables, except for interest rates, enter the models either as logarithms or first differences of their logs.

Figure 16. Short-term interest rates in Poland (quarterly data, %)



Source: Bloomberg, NBP.

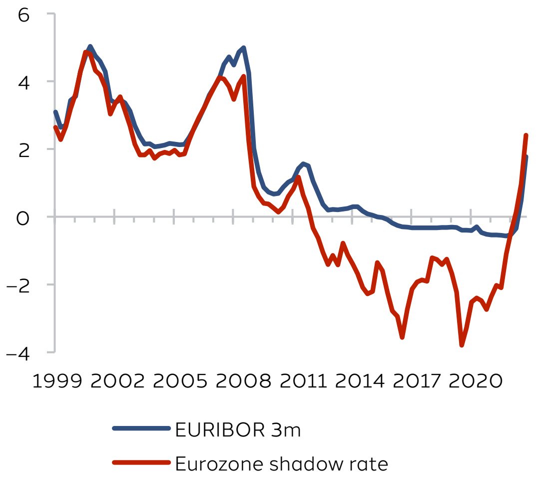
Figure 17. Long-term interest rates in Poland (quarterly data, %)



Source: Bloomberg.

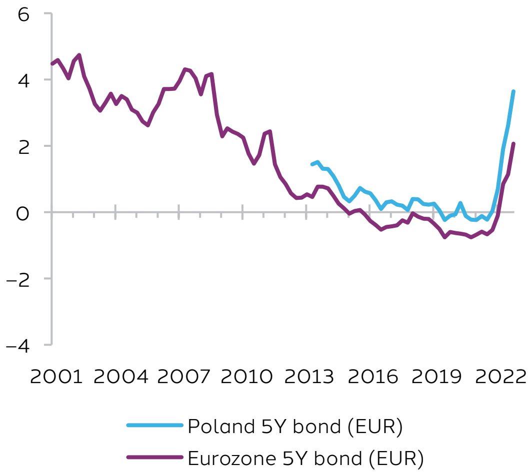
We use the 3-month WIBOR (Warsaw Interbank Offered Rate) as a popular short-term interest rate benchmark for the calculation of the natural interest rate in Poland (Figure 16). This money market rate, as it was subject to the ZLB in the period from 2020Q2 to 2021Q3, is replaced by the NBP estimates (Hertel et al. [2022]) of the shadow interest rate compiled with the method proposed by Krippner [2013]. As we treat interbank deposits as risk-bearing, non-liquid securities rather than risk-free ones, we also incorporate a policy rate (NBP reference rate) into the time-series models for the period when the NBP conducted conventional open-market operations. Lastly, we use 5-year Treasury bond interest rates denominated in PLN and EUR (Figure 17, Figure 19).

Figure 18. Short-term interest rates in the euro area (quarterly data, %)



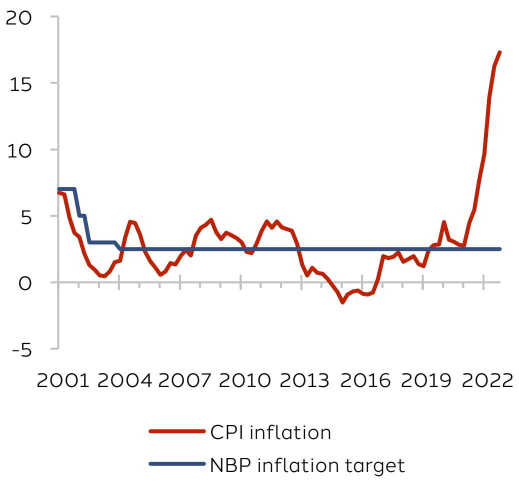
Source: https://www.ljkmfa.com/ as of January 2023, updated calculations based on Krippner [2013].

Figure 19. Long-term interest rates in Poland and in the euro area (government bonds denominated in euro, quarterly data, %)



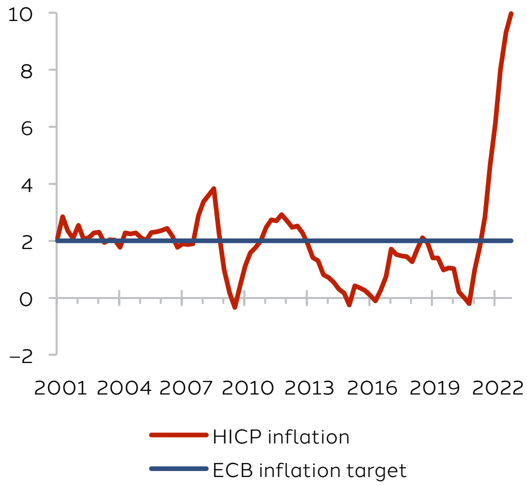
Source: Bloomberg.

Figure 20. Inflation in Poland (quarterly data, %, y-o-y)



Source: Statistics Poland, NBP.

Figure 21. Inflation in the euro area (quarterly data, %, y-o-y)



Source: Eurostat, ECB.

To disentangle nominal and real interest rate variability in the time-series models, we use the consumer price index (CPI) from Statistics Poland (Figure 20). Alternatively, we applied data on Narodowy Bank Polski's implicit and explicit inflation targets. In the beginning of the sample, as long as long-term interest rates in Poland were influenced by strong expectations on the disinflation process (until 2003Q2), we adjusted the term structure of interest rates to allow for a positive term premium.

In the open-economy setup, we expand the dataset to include foreign financial market interest rates and the bilateral PLNEUR exchange rate. In order to consistently capture the period of ECB unconventional monetary policy, the estimates of the natural interest rate are based on the policy rate from the Krippner webpage[16]. We use the shadow policy rate for the EA, which closely tracks the main policy interest rate during the period of ECB conventional monetary policy, but is more sensitive to market expectations (Figure 18). The dataset for the euro area consists of inflation rates (harmonised index of consumer prices, i.e. HICP from

16 See https://www.ljkmfa.com/visitors and https://www.ljkmfa.com/wp-content/uploads/2021/08/SSR_Estimates_20240828.xlsx (latest access date 25.09.2024).

Eurostat and the inflation target from the ECB, Figure 21) and long-term (5-year[17]) government bond yields denominated in euros for eurozone countries and Poland. Finally, we consider the fundamental equilibrium exchange rate (FEER) as the indicator of the exchange rate trend. We use the Global Value Chains extended FEER framework proposed by **Kuziemska-Pawlak and Mućk** [**2022**]. Compared to **Kuziemska-Pawlak and Mućk** [**2022**], unitary demand elasticities of trade flows are assumed.

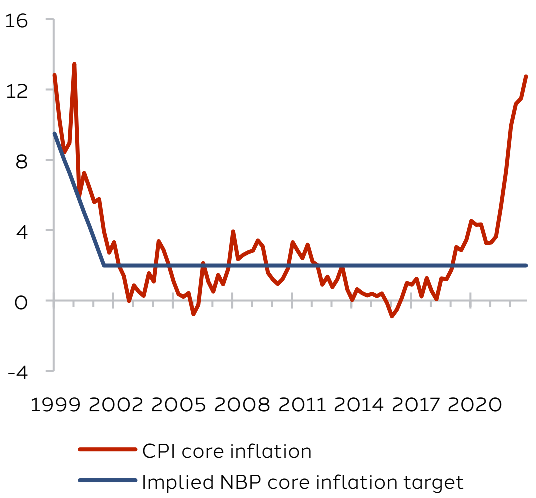
Semi-structural models

For estimating the semi-structural models, we use additional macroeconomic time series. Real GDP, measured in constant 2010 base-year prices for both PL and EA, was obtained from Eurostat. Capacity utilisation data for both PL and EA come from the OECD Business Tendency Surveys (Manufacturing): Capacity Utilisation: Rate of Capacity Utilisation indicators, accessed through the St. Louis Fed FRED database. The HICP core price index for the EA was sourced from Eurostat, while the CPI core price index for PL was taken from Statistics Poland.

The estimation starts in 1995Q1 and until 1998Q4 only GDP and the cyclical component of capacity utilisation are provided to help the procedure recover the initial growth rate of the GDP trend. All other time series are provided for the 1999–2022 period. Since the shadow rates are not directly observable, and both GDP and capacity utilisation measures are subject to statistical errors, in the case of these variables we admit the following measurement errors: 1 pp for the shadow rates in both PL and EA, 0.7 and 0.5 pp for GDP in PL and EA respectively, and 2.0 and 1.0 pp for capacity utilisation in PL and EA respectively (Figure 24, Figure 25).

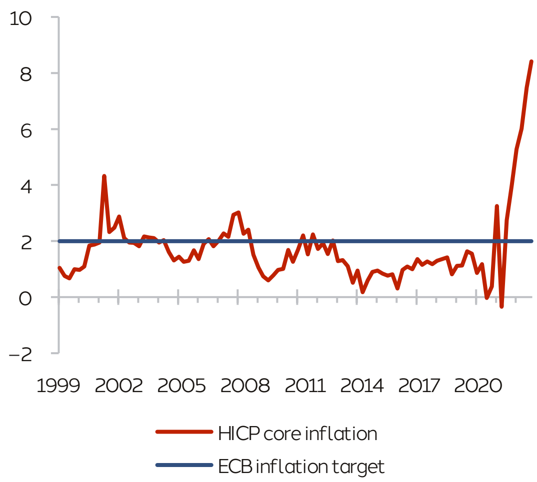
The BM method allows for specifying a time-varying inflation target. While we do not need to resort to this for the EA, the beginning of the sample in PL consists of a rapid disinflation process, during which the NBP inflation target was repeatedly adjusted (Figure 22, Figure 23). Since the results from the very beginning of our sample (1999–2003) are not crucial, we assume a linearly decreasing inflation target, stabilising at 2.5% starting from 2001Q3. Additionally, since in PL the average core inflation rate was lower than the headline by around 0.5 pp throughout our sample period, we assumed that in terms of core inflation the implied NBP inflation target was lower by 0.5 pp.

Figure 22. Core inflation in Poland (quarterly data, s.a. and annualized, %)



Sources: Statistics Poland, authors' own elaboration.

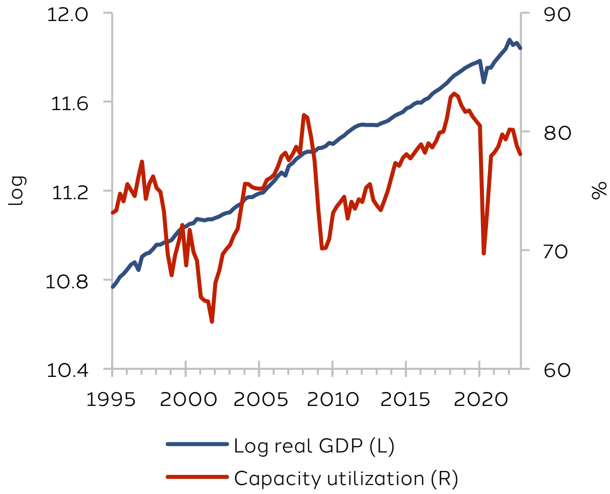
Figure 23. Core inflation in the euro area (quarterly data, s.a. and annualized, %)



Source: Eurostat, authors' own elaboration.

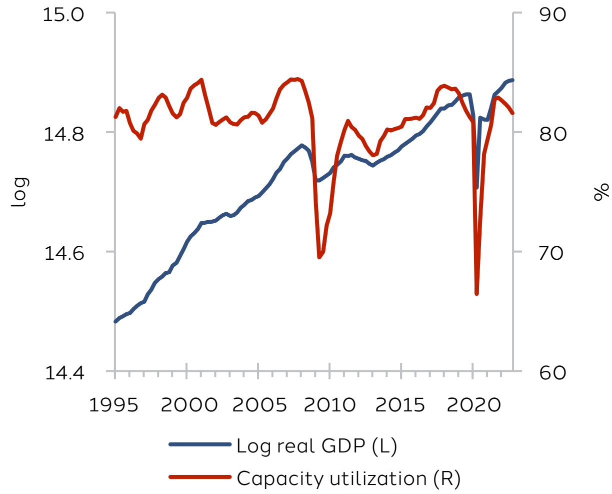
[17] In the paper, we use 5-year bonds for Poland and EA. In the working paper version of the paper, we also used 10-year bonds as a robustness test. See the Appendix in **Bielecki et al.** [**2023**] for an overview of these results.

Figure 24. Real GDP (logarithmic scale) and capacity utilization (%) in Poland (quarterly data)



Sources: Eurostat, authors' own elaboration.

Figure 25. Real GDP (logarithmic scale) and capacity utilization (%) in the euro area (quarterly data)



Sources: Eurostat, authors' own elaboration.

Structural model

Demographic data and projections come from the 2022 edition of the United Nations' World Population Prospects (WPP, Figure 26). The assumptions behind the evolution of TFP for the EA are based on data provided by the Area Wide Model database (AWM) and the European Commission's Ageing Working Group 2018 projections (AWG). The corresponding data for PL are constructed based on the Penn World Tables database version 10 (PWT). We assume that the TFP growth rate in PL will converge to the EA level by 2045, at which point the TFP gap will be nearly closed (Figure 27).

Additionally, we approximate the process of the gradual integration of PL in global financial and trade markets by a smoothly time-varying country risk premium multiplier (Figure 28), so that the model replicates the observed Net Foreign Assets evolution. The NFA data for PL for the 1995–2021 period were taken from the NBP (Figure 29).

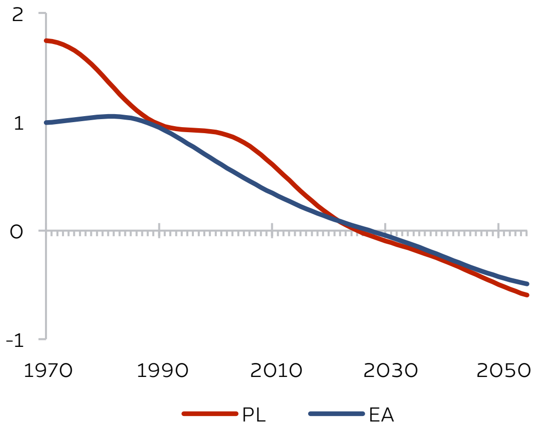
The model features an age-dependent productivity profile based on Household Finance and Consumption Survey data (HFCS) on household labour income for EA countries (Figure 30). Since we do not have corresponding data for PL, we assume an identical profile. Since the AWG projects starkly declining pension replacement rates (Figure 31) in Poland, we include them as an exogenous driver.

The model parameters were calibrated symmetrically for PL and the EA (Table 1). The households' discount rate was chosen so that the model real interest rate in the EA for the 1999–2008 period was equal to the data average.

Table 1. Calibrated BBK model parameters

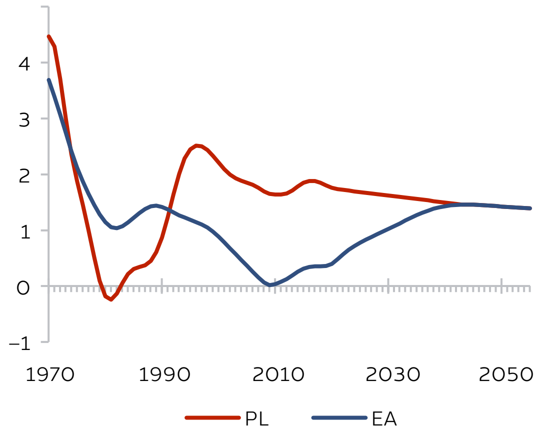
Parameter	Description	Value
ρ	Households' discount rate	−0.0065
J	Maximum household age	80
JR	Exogenous retirement age	43
α	Elasticity of output w.r.t. capital	0.25
δ	Capital depreciation rate	0.10
μ	Product markup	1.25
ξ	Country risk premium	0.025
G/Y	Government spending to GDP ratio	0.2
B/Y	Government debt to GDP ratio	0.6

Figure 26. Historical and projected adult population growth rates in Poland and in the euro area (quarterly data, %)



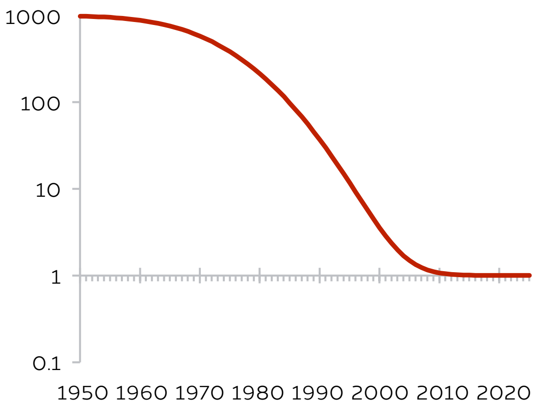
Sources: WPP, authors' own elaboration.

Figure 27. Historical and projected technological progress rates in Poland and in the euro area (quarterly data, %)



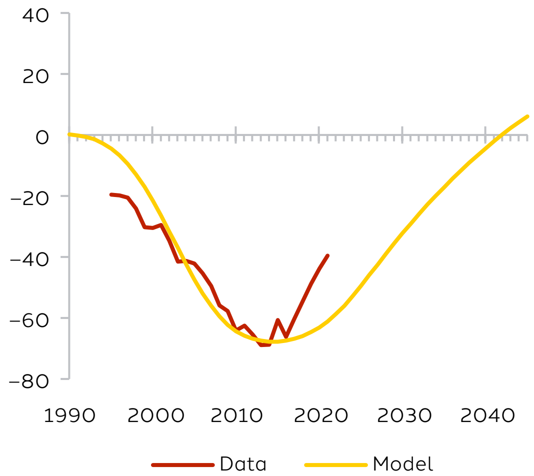
Sources: AWM, AWG, PWT, authors' own elaboration.

Figure 28. Time-varying risk premium multiplier for Poland (quarterly data, log scale)



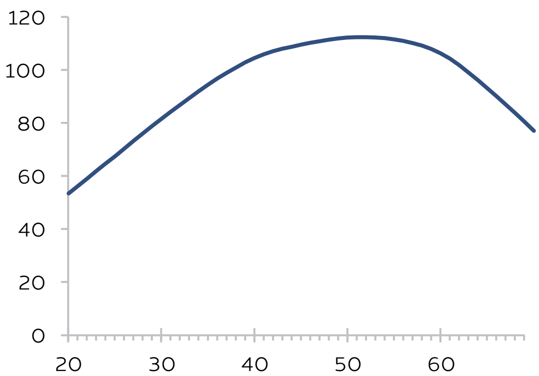
Source: Authors' own elaboration.

Figure 29. Net foreign assets position for Poland, data and model (quarterly data, % of GDP)



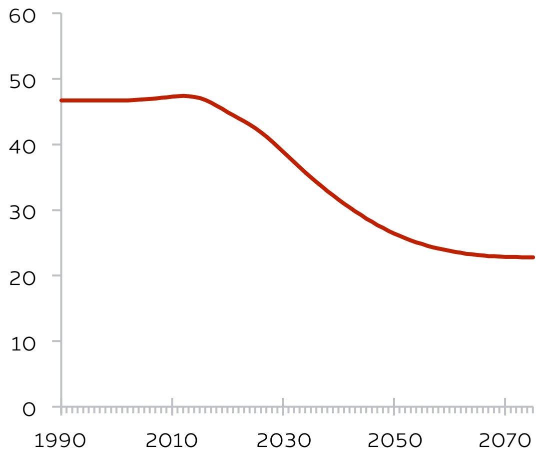
Sources: NBP, authors' own elaboration.

Figure 30. Age productivity profile (average life time productivity = 100)



Sources: HFCS, authors' own elaboration.

Figure 31. Pension replacement rate in Poland (quarterly data, %)



Sources: AWG, authors' own elaboration.

Method and additional results

Time-series models

The DGGT model (1-4) can be cast into a Bayesian framework and estimated using the Gibbs sampler after a prior elicitation procedure. The detailed two-step sampling method (with a simulation smoother necessary to obtain draws for the latent states) is described in the Appendix of Del Negro et al. [2017]. A relatively tight prior, $p(\Sigma_e)$, which is inverse-Wishart distributed, guarantees the smoothness of the common trends in $\bar{y}_t$ (Table 2). In turn, there is a relatively loose prior on $p(\Sigma_\varepsilon)$, with a standard Minnesota prior in the stationary VAR part using the indicator function, $\varphi = vec(\Phi)$, $p(\varphi|\Sigma_\varepsilon) = N(\underline{\varphi}, \Sigma_\varepsilon \otimes \underline{\Omega})I(\varphi)$. Indicator $I(\varphi)$ is equal to 0 if the characteristic roots of the VAR polynomial lie outside the unit circle, and 1 otherwise, which excludes the possibility of explosive paths in VAR.

Table 2. Hyperparameters on the random walks for common trends in the DGGT model

scale factor	$\bar{\pi}$	$\bar{r}$	$\overline{cy}$	$\overline{tp}$	$\bar{r}^{EA}$	$\overline{cy}^{EUR}$	$\bar{x}$	$\bar{\pi}^{EA}$
$\frac{1}{400}$	4.0	2.5	0.25	0.5	2.0	0.25	4.0	2.0

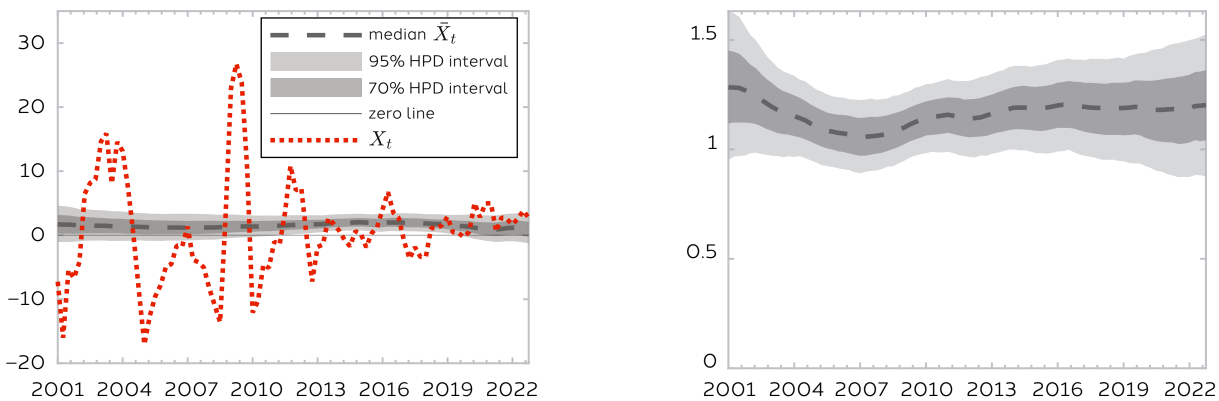
Note: The elements in the table are standard deviations of prior distributions on the diagonal of Σ_e; their squares are multiplied by the common scale factor.

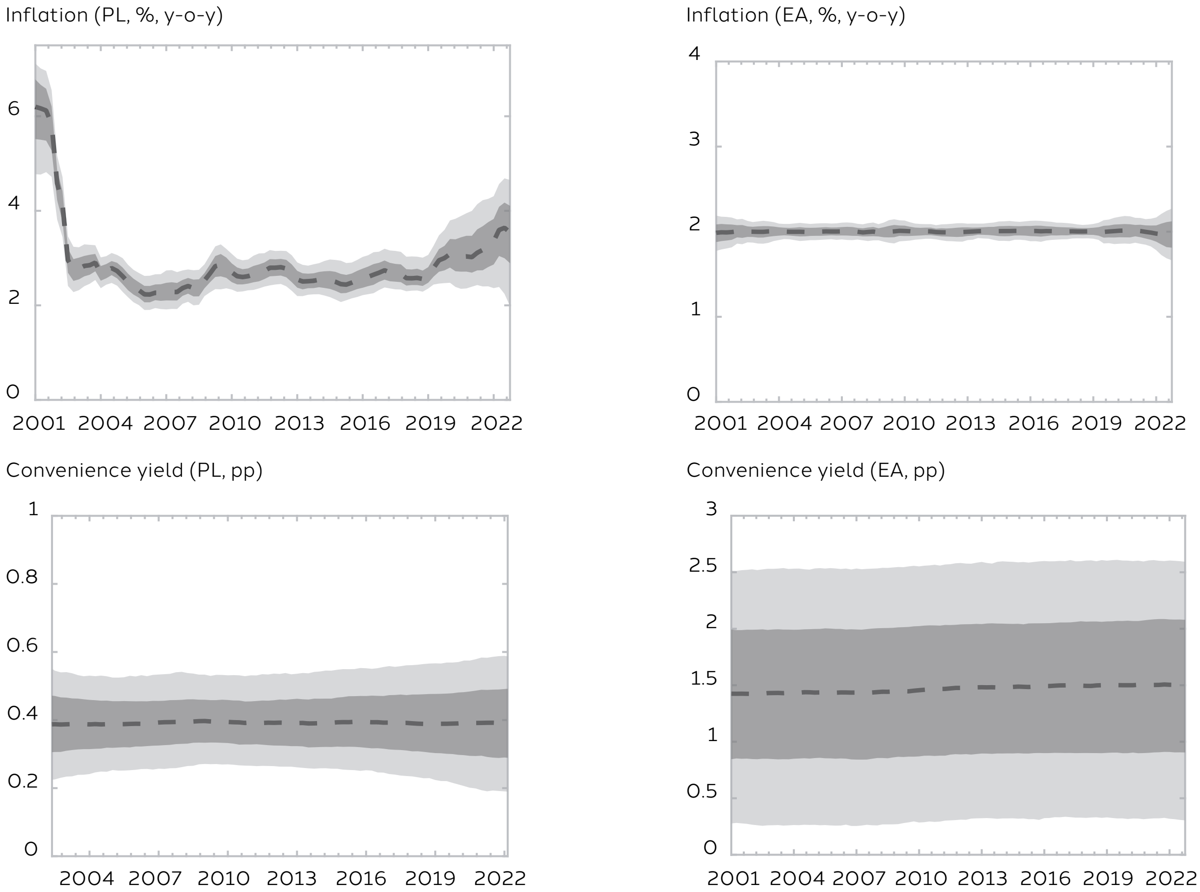
The initial conditions for normally distributed latent states: $\bar{y}_0$, $\tilde{y}_{0:-p+1}$ should be of less importance given the simulation smoother step if one omits a few initial observations from the posterior analysis as we do. Finally, we adjusted the Matlab procedures from Del Negro et al. [2017] available at https://github.com/FRBNY-DSGE/rstarBrookings2017 to draw 5,000 replications from the marginal posterior distribution for each of the models.

In the main text, we discuss the results of the baseline version of the DGGT model. Figure 32 presents the six remaining common trends (additionally to the NRI) and their uncertainty. For the EURPLN exchange rate, we accompany the trend with the observables to stress the difference in the scales of fluctuations in the persistent and stationary components.

Figure 32. The estimates of trend components (median, and 95% and 70% HPD intervals, quarterly data)

EURPLN (X_t, annualized pp) 5-year term premium (pp)



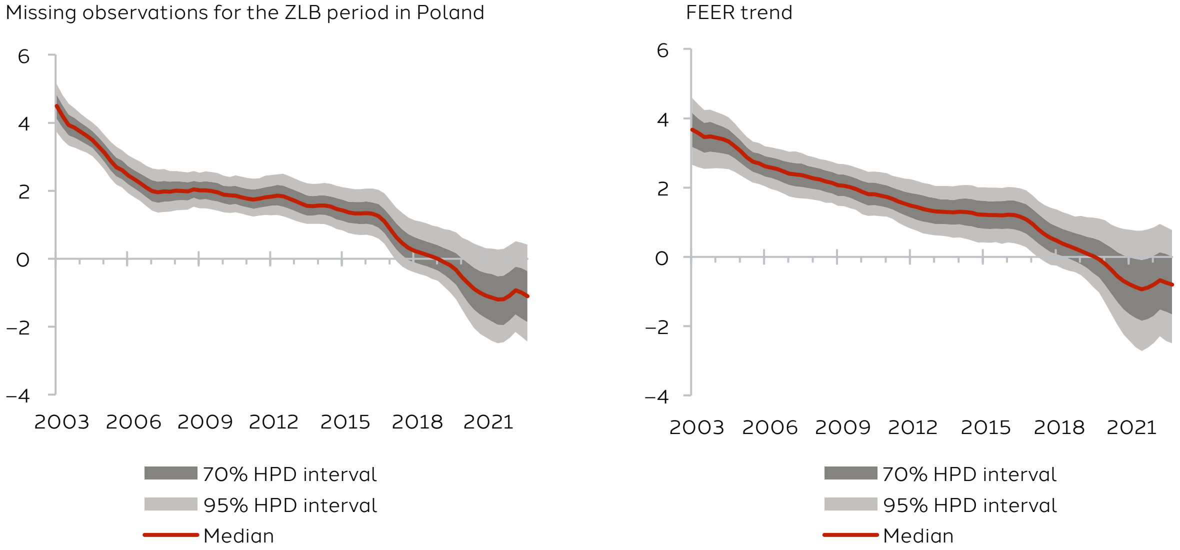


Source: Authors' own elaboration.

In the first alternative version of the open-economy DGGT model, we substitute the shadow rate estimates for Poland à la **Krippner** [**2013**] with missing observations for the ZLB period. Compared to the baseline model, the median estimates of the NRI increase by an average of 0.1 pp, particularly during and after the ZLB period (by 0.4 pp). However, there remains uncertainty about whether the NRI is negative toward the end of the sample period; zero falls within the 95% credibility interval but is outside the 70% interval. Paradoxically, the uncertainty of the NRI in 2022Q4 diminishes on one side and the 70% HPD interval is from –1.9 to –0.4 pp (left panel in Figure 33). This is likely due to the exceptionally negative Krippner-style shadow rates observed in Poland during the pandemic and the assumption of symmetrical deviations from the trend component. The median estimate of the NRI in the baseline version, however, is still within the 70% HPD interval of this alternative "missing observation" version.

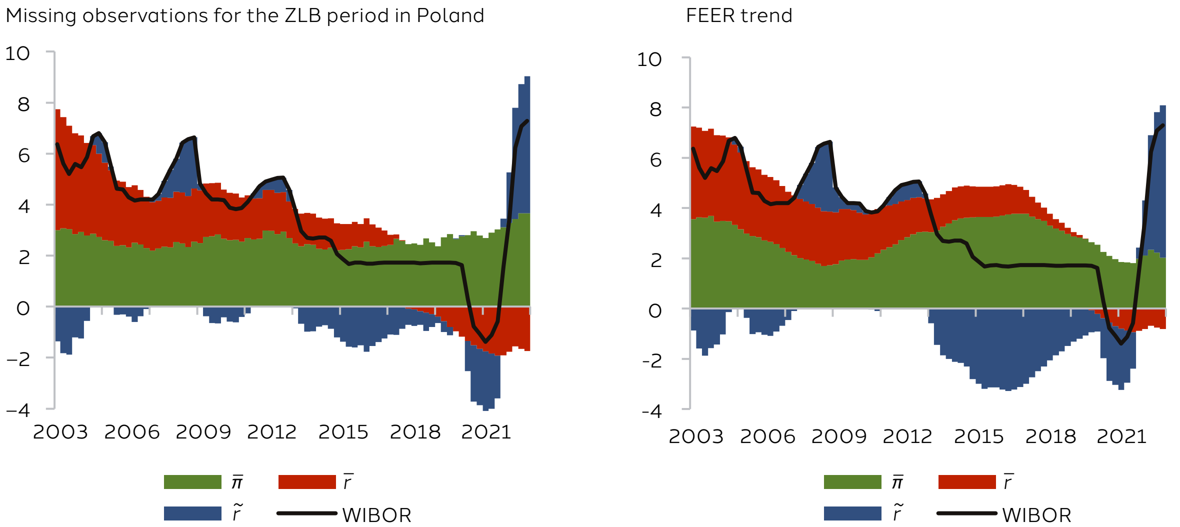
In the second alternative version of the open-economy DGGT model (FEER trend), we additionally use FEER to better account for trends in the exchange rates. The approach introduces some economic rationale behind the evolution of the exchange rate, which, probably due to the assumption of UIP, translates into less uncertainty about the NRI in Poland (right panel in Figure 33). The median estimate of the NRI from the baseline version is within the 70% HPD interval of the FEER version in the full sample, and at the end of the sample the NRI from FEER version is mildly negative (zero is on the edge of the upper bound of the 70% credible interval from 2022Q1 to 2022Q4, c.f. the right panel in Figure 33).

Figure 33. Estimates of the NRI in Poland in the alternative versions of open economy DGGT model (quarterly data, %)



Source: Authors' own elaboration.

Figure 34. Time-series decomposition of WIBOR in the alternative versions of time-series models (quarterly data, pp)



Source: Authors' own elaboration.

Table 3. Priors and posterior estimates of the parameters in the BM and HLW models

Param.	Prior			HLW PL		BM PL		HLW EA		BM EA	
	shape	mean	std. dev.	mean	95% HPD	mean	95% HPD	mean	95% HPD	mean	95% HPD
a_1	normal	0.00	1.00	0.88	0.39–1.41	0.91	0.44–1.40	0.52	0.30–0.75	0.51	0.29–0.73
a_2	normal	0.00	1.00	−0.20	−0.65–0.22	−0.26	−0.68–0.13	0.03	−0.18–0.24	0.03	−0.17–0.24
a_r	beta	0.10	0.05	0.06	0.01–0.11	0.09	0.03–0.16	0.05	0.00–0.11	0.12	0.02–0.22
b_1	normal	0.00	1.00	0.75	0.54–0.96	0.92	0.82–1.00	0.77	0.56–0.98	0.93	0.84–0.99
b_y	normal	0.00	1.00	0.38	0.12–0.66	0.38	0.15–0.60	0.11	0.02–0.21	0.11	0.02–0.21
c_0	normal	0.00	1.00	1.61	1.00–2.23	1.58	1.02–2.19	1.49	1.26–1.72	1.49	1.25–1.72
c_1	normal	0.00	1.00	0.30	−0.33–0.91	0.18	−0.38–0.74	0.72	0.50–0.94	0.75	0.53–0.98
ρ_1	normal	0.00	1.00			0.88	0.83–0.93			0.75	0.59–0.89

cont. Table 3

Param.	Prior			HLW PL		BM PL		HLW EA		BM EA	
	shape	mean	std. dev.	mean	95% HPD	mean	95% HPD	mean	95% HPD	mean	95% HPD
ρ_π	normal	0.00	1.00			1.43	0.80–2.09			0.63	0.19–1.11
ρ_y	normal	0.00	1.00			2.38	1.14–3.62			0.66	0.19–1.11
$\sigma_{\tilde{y}}$	inv. gamma	1.00	Inf	0.80	0.44–1.14	0.81	0.47–1.14	1.39	1.16–1.62	1.35	1.12–1.58
$\sigma_{\bar{y}}$	inv. gamma	1.00	Inf	0.74	0.42–1.06	0.72	0.39–1.02	0.42	0.25–0.58	0.47	0.31–0.63
σ_π	inv. gamma	1.00	Inf	1.24	1.06–1.44	1.27	1.08–1.46	0.78	0.67–0.89	0.75	0.64–0.86
σ_r / σ_i	inv. gamma	1.00	Inf	0.80	0.69–0.92	0.71	0.58–0.85	0.44	0.36–0.51	0.29	0.21–0.38
σ_{cu}	inv. gamma	1.00	Inf	0.46	0.23–0.76	0.47	0.23–0.77	0.34	0.20–0.49	0.34	0.20–0.49
σ_g	inv. gamma	0.10	Inf	0.07	0.02–0.15	0.08	0.02–0.16	0.09	0.03–0.15	0.07	0.03–0.12
σ_z	inv. gamma	0.30	0.10	0.40	0.11–0.89	0.42	0.17–0.77	0.42	0.11–0.92	0.38	0.22–0.57

Source: Authors' own elaboration.

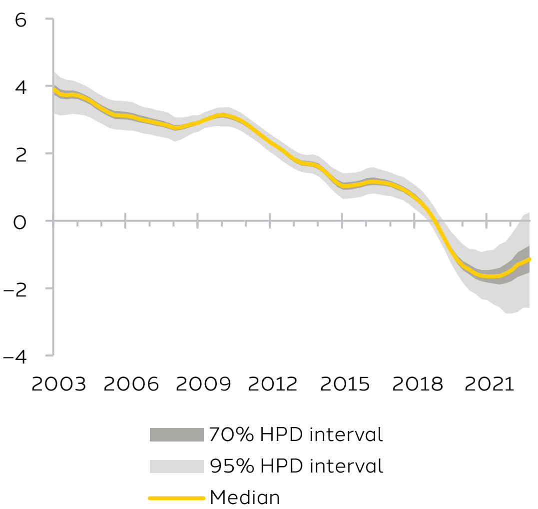
Semi-structural models

The majority of parameter priors were designed to be only weakly informative, imposing a standardised normal distribution (Table 3). The single exception was the a_r parameter in the IS curve, where, due to the pile-up problem, a beta prior with the mean of 0.1 and standard deviation of 0.05 was imposed. The priors for the standard deviation of shocks follow the inverse gamma distribution with standard deviations on g and z based on the HLW model estimated on the pre-pandemic EA sample. Both the BM and HLW models were estimated using 2 million MCMC draws with an acceptance rate of around 0.25.

As a robustness check, we re-estimate the BM and HLW models for both monetary areas where, instead of using shadow interest rates, we use missing observations for the ZLB periods. The results for Poland and for the euro area in the HLW model are almost identical to the baseline.

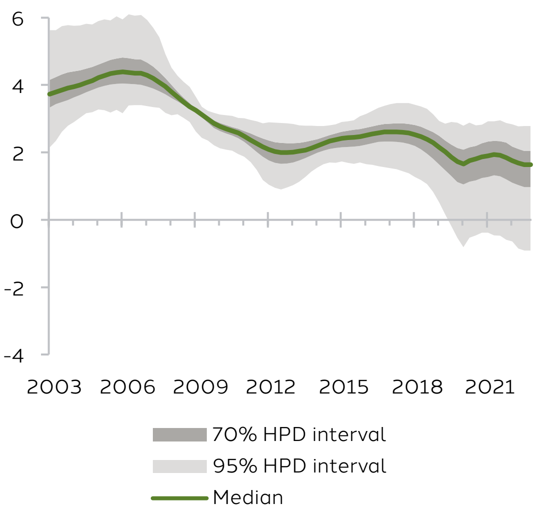
The BM model uncovers a markedly different trajectory for the euro-area NRI between the financial crisis and the pandemic, which only gradually declines from around 0.7% in 2007Q1 to around –1.0% in 2014Q1 (compare to the baseline estimates of 1.6% in 2007Q1 and –3.1% in 2014Q1), and continues an almost uniform decline towards its trough of –3.1% in 2021Q1. The model-inferred interest rates over the 2012–2019 period are close to the EURIBOR 3M values, and so the model is effectively oblivious to the additional monetary policy easing implemented via various asset purchase programs.

Figure 35. Alternative estimates of the NRI in Poland, BM model (quarterly data, %)



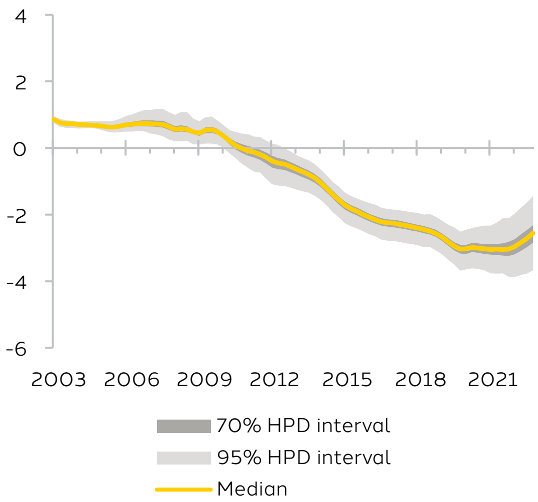
Source: Authors' own elaboration.

Figure 36. Alternative estimates of the NRI in Poland, HLW model (quarterly data, %)



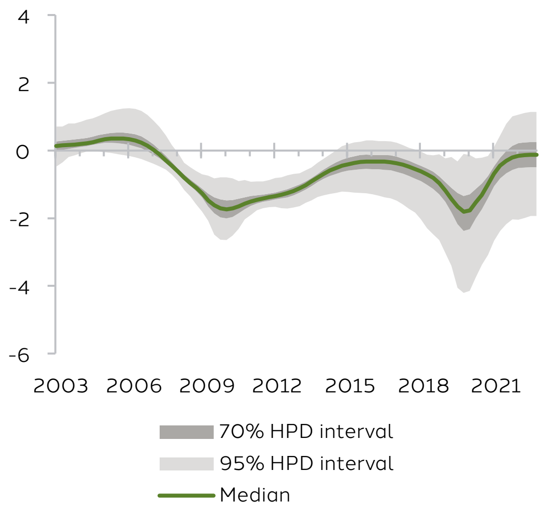
Source: Authors' own elaboration.

Figure 37. Alternative estimates of the NRI in the euro area, BM model (quarterly data, %)



Source: Authors' own elaboration.

Figure 38. Alternative estimates of the NRI in the euro area, HLW model (quarterly data, %)



Source: Authors' own elaboration.

Structural model

The BBK model is solved using a fully nonlinear, deterministic solver available in Dynare. Below we reproduce the full set of model equilibrium conditions that jointly determine the evolution of real per capita allocations and real prices, for given initial conditions and for given paths of exogenous deterministic variables: growth rate of 20-year-olds $n_{1,t}$, mortality rate $\omega_{j,t}$ $(j = 1, \ldots, J-1)$, labour productivity profile z_j, TFP gap $A_t \leq 1$, rate of technological progress x_t, pension replacement rate ϱ_t, country risk premium multiplier γ_t, and world real interest rate r_t^*.

Households

$$c_{j,t} + (1 + x_{t+1}) a_{j+1,t+1} = (1_{j<JR}) \left(1 - \tau_t + \frac{\pi_t}{w_t h_t}\right) w_t z_j + (1 - \tau_t)(1_{j \geq JR}) pen_t + beq_t + (1 + r_t) a_{j,t}$$

$$a_{0,t} = 0$$

$$a_{J,t} = 0$$

$$(1 + x_{t+1}) c_{j+1,t+1} = \beta (1 - \omega_{j,t})(1 + r_{t+1}) c_{j,t}$$

Demographics

$$n_{1,t+1} = \frac{N_{1,t+1}}{N_{1,t}} - 1$$

$$N_{j+1,t+1} = (1 - \omega_{j,t}) N_{j,t}$$

$$N_t = \sum_{j=1}^{J} N_{j,t}$$

$$n_{t+1} = \frac{N_{t+1}}{N_t} - 1$$

Aggregation over households

$$c_t = \sum_{j=1}^{J} \frac{N_{j,t} c_{j,t}}{N_t}$$

$$h_t = \sum_{j=1}^{JR-1} \frac{N_{j,t} z_j}{N_t}$$

$$a_{t+1} = \sum_{j=1}^{J} \frac{N_{j,t} a_{j+1,t+1}}{N_{t+1}}$$

$$beq_t = \sum_{j=1}^{J} \frac{\left(N_{j,t-1} - N_{j,t}\right)\left(1 + r_t\right) a_{j,t}}{N_t}$$

Firms

$$\left(1 + n_{t+1}\right)\left(1 + x_{t+1}\right) k_{t+1} = \left(1 - \delta\right) k_t + i_t$$

$$r_t = \frac{\alpha}{\mu} A_t k_t^{\alpha-1} h_t^{1-\alpha} - \delta$$

$$w_t = \frac{1-\alpha}{\mu} A_t k_t^{\alpha} h_t^{-\alpha}$$

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$$

$$\pi_t = y_t - w_t h_t - i_t$$

Government

$$pen_t = \varrho_t w_t \frac{\sum_{j=1}^{JR-1} N_{j,t} z_j}{\sum_{j=1}^{JR-1} N_{j,t}}$$

$$\tau_t w_t \sum_{j=1}^{JR-1} \frac{N_{j,t}}{N_t} z_j + \left(1 + n_{t+1}\right)\left(1 + x_{t+1}\right) \frac{B}{Y} = \frac{G}{Y} + \left(1 - \tau_t\right) pen_t \sum_{j=JR}^{J} \frac{N_{j,t}}{N_t} + \left(1 + r_t\right) \frac{B}{Y}$$

External sector

$$1 + r_{t+1} = \left[1 + \gamma_t \xi \left(\exp\left(-b_t^* / y_t\right) - 1\right)\right]\left(1 + r_{t+1}^*\right)$$

Market clearing

$$a_t = k_t + b_t + b_t^*$$

$$\left(1 + n_{t+1}\right)\left(1 + x_{t+1}\right) b_{t+1}^* = \left(1 + r_t\right) b_t^* + y_t - c_t - i_t - \frac{G}{Y}$$