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Long Shadows of Financial Shocks: An Endogenous Growth Perspective*

Długie oddziaływanie szoków finansowych z perspektywy wzrostu endogenicznego

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Abstract

The Great Recession has resulted in a seemingly permanent level shift in many macroeconomic variables. This paper presents a microfounded general equilibrium model featuring frictional labour markets and financial frictions that is able to replicate the business cycle features of establishment dynamics and generate the procyclicality of R&D expenditures. This makes it possible to demonstrate the channels through which productivity and financial shocks influence the aggregate endogenous growth rate of the economy, creating level shifts in its balanced growth path. The results indicate that financial shocks are an important driver of aggregate fluctuations and their influence is especially pronounced for establishment entry. Since the growth rate of the economy can in principle be affected by policy measures, the macroeconomic and welfare effects of applying several subsidy schemes are examined. Subsidising R&D expenditures and lowering barriers to entry were found to be welfare improving, in line with endogenous growth literature. At odds with this literature, static subsidies to incumbents' operating costs were also found to be welfare improving, a result that only emerges under the stochastic setting.

Słowa kluczowe:
cykle koniunkturalne, dynamika przedsiębiorstw, endogeniczny wzrost, kapitał obrotowy, szoki finansowe

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Streszczenie

Wielka recesja poskutkowała długotrwałym przesunięciem w dół przebiegu wielu zmiennych makroekonomicznych. Artykuł przedstawia wyprowadzony z mikropodstaw model równowagi ogólnej uwzględniający niedoskonałości rynków pracy i rynków finansowych. Model jest w stanie odzwierciedlić procykliczność wydatków na badania i rozwój oraz cykliczne właściwości dynamiki przedsiębiorstw. Pozwala to na uchwycenie kanałów, przez które szoki produktywności i szoki finansowe wpływają na endogeniczne tempo wzrostu oraz skutkują przesunięciami ścieżki zrównoważonego wzrostu gospodarki. Wyniki wskazują, że szoki finansowe są ważną siłą napędową zagregowanych wahań, a ich wpływ jest szczególnie wyraźny w przypadku dynamiki wejść nowych przedsiębiorstw. Ponieważ polityka gospodarcza może wpływać na tempo wzrostu gospodarczego, badane są

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makroekonomiczne i dobrobytowe skutki stosowania kilku typów subsydiów. Zgodnie z wynikami wcześniejszych badań opisanymi w literaturze dotyczącej wzrostu endogenicznego subsydiowanie wydatków na badania i rozwój oraz obniżanie barier wejścia na rynek poprawia dobrobyt. W niniejszym artykule – inaczej niż we wspomnianych badaniach – stwierdzono, że subsydiowanie bieżących kosztów stałych przedsiębiorstw również poprawia dobrobyt, a wynik ten można uzyskać wyłącznie poprzez uwzględnienie wpływu zaburzeń stochastycznych.

Introduction

The experience of the Great Recession and its aftermath has compelled many macroeconomists to examine the links between the financial sector and real activity. This paper presents a model of heterogeneous, monopolistically competitive establishments that endogenously choose the intensity of research and development. The model also features endogenous entry and exit, and incorporates search and matching frictions in the labour market, as well as a reduced form of the financial shock. The paper brings together several strands of literature on business cycles, firm dynamics and endogenous growth and carries important policy implications for industrial policy over the business cycle.

The two main mechanisms that generate volatile and procyclical R&D expenditures are an increased willingness of incumbents to invest in R&D in good times, and procyclical entry rates. These regularities imply that the endogenous growth rate of the economy is also procyclical, giving rise to hysteresis effects, where the balanced growth path permanently shifts in response to transitory shocks. As a consequence, the welfare effects of business cycles are much higher than for the exogenous growth models, as consumption is not only volatile but also subject to level effects.

The results from the model indicate that the cost of business cycle fluctuations is two orders of magnitude higher than in the exogenous growth variant of the model. The ability to potentially affect the growth rates and volatility of the economy through appropriate industrial policy creates ample space for policy intervention via static and countercyclical subsidies. The most positive welfare effect is achieved through countercyclical subsidies to incumbents' operating cost, as it prevents excessive exits and encourages more R&D spending. Accounting for frictions in the labour market results in welfare gains from static subsidies to incumbents' operating cost, a result at odds with the endogenous growth models that abstract from this friction.

The paper is based on the neo-Schumpeterian endogenous growth paradigm, pioneered by **Grossman and Helpman [1991]** and **Aghion and Howitt [1992]**. Following the seminal contribution by **Klette and Kortum [2004]**, there is a growing literature on the relationship between endogenous growth and firm dynamics. This paper is close in spirit to work **Acemoglu et al. [2018]** who study the consequences of subsidy schemes for R&D expenditures and growth, and related works include **Acemoglu and Cao [2015]** and **Akcigit and Kerr [2018]**. The common assumption in those papers is that the incumbent firms innovate on their own products in a neo-Schumpeterian quality-ladder set-up. I contribute to that literature by considering similar underlying mechanisms in a stochastic set-up, and I am able to analyse the effect of countercyclical subsidies.

The paper also belongs to the growing body of the literature concerned with firm level heterogeneity and dynamics. **Bartelsman and Doms [2000]** provide a review of the early literature focused on documenting productivity differences and growth across firms and linking those phenomena to aggregate outcomes. **Foster et al. [2001]** emphasise the role of cyclical entry for aggregate productivity growth. The role of entry and exit channels for macroeconomic dynamics has been recognised and studied by **Hopenhayn [1992]**, **Devereux et al. [1996]**, **Campbell [1998]**, **Jaimovich and Floetotto [2008]**, **Bilbiie et al. [2012]**, **Chatterjee and Cooper [2014]** and **Lee and Mukoyama [2015]**, although none of those works incorporate the full set of firm dynamics considered here. **Gourio et al. [2016]** show, using regional US data, that low entry rates in the 2007–2009 period contributed significantly to low employment and labour productivity growth. **Clementi and Palazzo [2016]** study full firm dynamics over the business cycle, although their analysis focuses on the firm-level investment in physical capital, rather than innovation, which is the core mechanism of this paper.

The model also features a frictional labour market subject to the search and matching friction in the tradition of **Diamond** [**1982**] and **Mortensen and Pissarides** [**1994**]. I follow an approach proposed by **Gertler and Trigari** [**2009**] that assumes convex hiring costs and is remarkably successful in replicating the labour market dynamics.

Financial frictions are modelled by assuming the working capital requirement as in **Christiano et al.** [**2010**] and having a reduced form of financial shocks in the form of the spread between the deposit and lending interest rates. There already exists literature that recognises the impact of financial disturbances on macroeconomic variables and firm dynamics, especially in the context of the Great Recession. **Estevao and Severo** [**2014**] use industry-level panel data from Canada and the United States to show that increases in the cost of funds for firms have negative effects on TFP growth. **Fernandez-Corugedo et al.** [**2011**] build a DSGE model with multiple components of the working capital channel and find that even under flexible prices a disruption to the supply of credit has large and persistent effects on the real economy. **Siemer** [**2014**] finds that tight financial constraints during the Great Recession were responsible for both low employment growth and firm entry rates. **Christiano et al.** [**2015**] build a medium-scale DSGE model to quantify the importance of several shocks during the Great Recession and find that financial frictions were driving a significant part of the macroeconomic variables' behaviour. **Ates and Saffie** [**2021**] build a small open economy entry-driven endogenous growth model to analyse the effects of a sudden stop using Chilean plant-level data. They find that entrants succeeding during financial shortage periods, while more productive on average, are substantially less numerous, generating a permanent loss of output and significant welfare costs. **Queralto** [**2020**] proposes a model where firm entry is hampered in times of financial stress, resulting in a lasting downward level shift in the economy's TFP, but his model does not feature the channel of innovation intensity. This paper builds on the existing literature by considering the effects of financial friction within a model that simultaneously accounts for rich firm dynamics, entry and exit decisions, and endogenous growth.

The remainder of the paper is organised as follows. The next section describes the model, focusing on presenting the problem of incumbents and potential entrants, and on describing the labour market and financial frictions. The third section provides a description of the data sources, parameter values and the estimation procedure, and discusses the cyclical properties of the model economy against the data. The fourth section applies the model to quantify the relative importance of the shocks during the Great Recession. It also analyses in depth the welfare properties of the model economy and presents the macroeconomic and welfare effects of applying several subsidy schemes, both static and counterfactual. The last section concludes.

Model

The model presented here is based on **Bielecki** [**2017**] and **Bielecki** [**2022**], but introduces two key changes. First, to capture the effects of the time-varying risk premium, a wedge between the deposit and lending interest rates is introduced. Second, as in **Christiano et al.** [**2010**], the model features the working capital channel, which was found to be an important amplifier of financial shocks to the real economy.

Households

The mass of representative households is normalised to unity. Each of the households is composed of a large family of workers who differ with respect to their employment status and skill level. Nevertheless, due to within-family risk sharing, all individuals enjoy identical levels of consumption. The representative household maximises the lifetime expected utility:

$$U_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \tag{1}$$

where c_t denotes per capita consumption, β represents the discount factor, and θ is the inverse of the elasticity of intertemporal substitution.

Following **Acemoglu et al.** [**2018**], it is assumed that workers belong to either of the two skill groups: unskilled workers of aggregate mass $(1 - s)$ supply labour to the production sector, while skilled workers of aggregate mass s are hired as managers or to perform research and development activities. Furthermore, within a time period, an individual worker may be employed and receive wage income, or unemployed and receive unemployment benefits. The budget constraint of a representative household is given by:

$$c_t + d_{t+1} = (1-s)\left[w_t^u n_t^u + b_t^u\left(1-n_t^u\right)\right] + s\left[w_t^s n_t^s + b_t^s\left(1-n_t^s\right)\right] + \left(1+r_t^d\right)d_t + t_t$$

where d_{t+1} is the end-of-period t stock of deposits supplied to the financial sector which yields interest at deposit rate r_t^d, w_t^u and w_t^s represent wage income of employed unskilled and skilled workers, respectively, while n_t^u and n_t^s are the employment rates, and b_t^u and b_t^s denote unemployment benefits. Finally, t_t represents the sum of all dividend payments and lump-sum transfers net of taxes received by the households.

The intertemporal optimisation by households yields the Euler equation:

$$c_t^{-\theta} = \beta \mathrm{E}_t\left[\left(1+r_{t+1}^d\right)c_{t+1}^{-\theta}\right] \tag{2}$$

and it is also convenient to define the stochastic discount factor of the households, which will be applied by firm managers to discount future profit streams:

$$\Lambda_{t,t+1} = \mathrm{E}_t\left[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\theta}\right] \tag{3}$$

Producers

The perfectly competitive final goods producers purchase differentiated intermediate goods varieties and transform them into final goods via the CES aggregator:

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}}$$

where Y_t is the aggregate final goods output, M_t is the mass of active intermediate goods producing establishments, $y_t(i)$ is the quantity demanded from the i-th producer, and σ is the elasticity of substitution between the varieties.

The monopolistically competitive intermediate goods first have to bear a fixed cost of operation f_t, which represents the costs of management and other non-production activities. Subsequently they can produce by employing capital and unskilled labour services according to the following Cobb-Douglas function:

$$y_t(i) = Z_t k_t^p(i)^\alpha\left[q_t(i)n_t^p(i)\right]^{1-\alpha}$$

where Z_t denotes the stochastic aggregate TFP parameter, $k_t^p(i)$ and $n_t^p(i)$ are the employed capital and unskilled labour respectively, and $q_t(i)$ represents the idiosyncratic quality level of the i-th variety at time period t. Parameter α describes the elasticity of intermediate goods output with respect to capital.

The intermediate goods producers choose such a combination of capital and labour that minimises their costs. As in **Christiano et al.** [**2010**], each producer has to finance a constant fraction ζ of both the capital rental cost and wage bill in advance of production by borrowing necessary funds at lending rate r_t^l.

As in Melitz [2003], the aggregate quality index Q_t summarises the aggregate situation in the intermediate goods sector as if it was populated by mass M_t of producers each with the same quality level. The index is constructed by applying the following formula:

$$Q_t = \left[\int_0^\infty q^{(1-\alpha)(\sigma-1)} \mu_t(q) \mathrm{d}q\right]^{\frac{1}{(1-\alpha)(\sigma-1)}}$$

where $\mu_t(q)$ denotes period t distribution of the idiosyncratic quality levels.

The aggregate final goods output can then be expressed as:

$$Y_t = M_t^{\frac{1}{\sigma-1}} Z_t \left(K_t^p\right)^\alpha \left(Q_t N_t^p\right)^{1-\alpha} \tag{4}$$

where K_t^p and N_t^p denote, respectively, the aggregate employment of capital and unskilled labour in the production sector, while the presence of M_t in the expression results from the love-for-variety phenomenon. In the long run, the only source of economic growth is the continued increase in the aggregate quality level over time, and both the capital stock and output will grow at the corresponding rate.

Incumbents

The intermediate goods producers can engage in research and development activities to have a chance at increasing their products' quality. It is useful to first define the relative quality level of the i-th variety as:

$$\phi_t(i) = \left(q_t(i) / Q_t\right)^{(1-\alpha)(\sigma-1)}$$

Its next period level will be decided by the following lottery:

$$\phi_{t+1}(i) = \begin{cases} \iota\phi_t(i)/\eta_t & \text{with probability } \chi_t(i) \\ \phi_t(i)/\eta_t & \text{with probability } 1-\chi_t(i) \end{cases}$$

where ι denotes the size of an innovative step and η_t is the transformed rate of growth of the aggregate quality index, which individual producers take as given:

$$\eta_t = \left(Q_{t+1} / Q_t\right)^{(1-\alpha)(\sigma-1)}$$

Innovative success probability $\chi_t(i)$ is chosen endogenously by each producer and is an increasing function of engaged R&D resources. The particular form of the success probability function is based on Pakes and McGuire [1994] and Ericson and Pakes [1995]:

$$\chi_t(i) = \frac{a \cdot rd_t(i)}{1 + a \cdot rd_t(i)}$$

where a is a parameter that describes the efficacy of R&D input $rd_t(i)$. The R&D process requires hiring both capital and skilled labour:

$$rd_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t \phi_t(i)}$$

where $k_t^x(i)$ and $n_t^x(i)$ denote the employment of capital and skilled labour in R&D activities.

The presence of the aggregate and relative quality levels in the expression lends itself to a very intuitive interpretation. The aggregate quality level in the numerator multiplies with R&D workers as they have access to a pool of common knowledge. However, over time, it is harder to come up with new ideas unless more resources are committed to R&D activities, which is captured by the aggregate quality level in the denominator. Finally, the presence of the relative quality level in the denominator represents the catch-up and headwind effects, depending on the establishments' relative position in the quality distribution.

The solution of the cost minimisation problem results in the following expression for the marginal cost of R&D activities:

$$mc_t^x(i) = \left(1+\zeta r_t^l\right) Q_t^{\alpha} \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha} \phi_t(i) \equiv \left(1+\zeta r_t^l\right)\left(\omega_t Y_t\right)\phi_t(i) \tag{5}$$

where $\tilde{w}_t^s$ denotes the skilled wage paid to the employment agency and $\omega_t Y_t$ captures the skilled marginal cost component common to all establishments.

To simplify the setup of the intermediate goods producer, managerial activities are assumed to require an identical combination of capital and skilled labour as R&D activities, and the fixed cost of operation becomes:

$$f_t = \left(1+\zeta r_t^l\right)\left(\omega_t Y_t\right) f$$

The real profit of an incumbent can then be expressed as the following function, affine in terms of the relative quality level:

$$\pi_t(i) = Y_t\left[\left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t(i)}{1-\chi_t(i)}\right)\phi_t(i) - \left(1+\zeta r_t^l\right)\omega_t f\right] \tag{6}$$

All producers with the same relative quality levels will behave identically. The value of a producer with relative quality level ϕ_t can be expressed as follows:

$$V_t(\phi_t) = \max_{\chi_t\in[0,1]}\left\{\pi_t(\phi_t,\chi_t) + \mathrm{E}_t\left[\Lambda_{t,t+1}\left(1-\delta_t\right)V_{t+1}\left(\phi_{t+1} \mid \phi_t,\chi_t\right)\right]\right\}$$

where δ_t is the endogenous probability that a producer will receive a death shock.

As the aggregate quality level trends upwards over time, causing other variables to trend as well, the following stationarisation is applied:

$$v_t(\phi_t) \equiv V_t(\phi_t)/Y_t = \max_{\chi_t\in[0,1]}\left\{\frac{\pi_t(\phi_t,\chi_t)}{Y_t} + \mathrm{E}_t\left[\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)v_{t+1}\left(\phi_{t+1} \mid \phi_t,\chi_t\right)\right]\right\}$$

Since the stationarised value function is a weighted sum of current and future profit flows, all of which are affine in ϕ_t, then the stationarized value function is also affine in ϕ_t and the following functional form can be imposed:

$$A_t + B_t\phi_t = \max_{\chi_t\in[0,1]}\left\{\begin{array}{c}\left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right)\phi_t - \left(1+\zeta r_t^l\right)\omega_t f \\ +\mathrm{E}_t\left[\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(A_{t+1} + B_{t+1}\frac{\chi_t(\iota-1)+1}{\eta_t}\phi_t\right)\right]\end{array}\right\} \tag{7}$$

where A_t and B_t are state-dependent coefficients that fluctuate over the business cycle.

The first order and envelope conditions of those producers can be then stated as follows:

$$0 = -\left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{1}{\left(1-\chi_t\right)^2} + \mathrm{E}_t\left[\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(B_{t+1}\frac{(\iota-1)}{\eta_t}\right)\right] \tag{8}$$

$$B_t = \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\right) + \mathrm{E}_t\left[\Lambda_{t,t+1}\left(1-\delta_t\right)\left(\frac{Y_{t+1}}{Y_t}\right)B_{t+1}\frac{\chi_t(\iota-1)+1}{\eta_t}\right] \tag{9}$$

As the relative quality level drops out of the above optimality conditions, one can conclude that all producers with a high enough relative quality level will choose the same success probability, and their size and growth rate will be uncorrelated, as postulated by Gibrat's law. This also implies that the ergodic distribution of the relative quality levels converges in the upper tail to the Pareto distribution with power parameter equal to 1[1].

However, the above representation abstracts from the case of producers with a negative continuation value who exit at the end of period and optimally choose not to engage in R&D activities at all. For the sake of tractability, it is assumed that all establishments above a certain threshold behave as described above, while all establishments below the threshold decide to exit and their stationarised value function is given by:

$$v_t(\phi_t) = \frac{1}{\sigma M_t}\phi_t - (1+\zeta r_t^l)\omega_t f \tag{10}$$

The threshold level of relative quality ϕ_t^* can therefore be found by comparing the two forms of the stationarised value functions:

$$(1+\zeta r_t^l)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\phi_t^* = \mathrm{E}_t\left[\Lambda_{t,t+1}(1-\delta_t)\left(\frac{Y_{t+1}}{Y_t}\right)\left(A_{t+1} + B_{t+1}\frac{\chi_t(\iota-1)+1}{\eta_t}\phi_t^*\right)\right] \tag{11}$$

Assuming that the quality distribution is a Pareto distribution over the entirety of its support yields a closed form expression for the mass of the exiting producers:

$$M_t^x = M_t(1-\chi_{t-1})\left(1-\frac{\phi_{t-1}^*}{\phi_t^*\eta_{t-1}}\right) \tag{12}$$

Entrants

The mass of potential entrants is unbounded although it will be pinned down by the equilibrium conditions. Similar to incumbents, they engage in R&D activities, although in this case the successful outcome of the innovation process results in entry rather than an improvement over the existing product.

The entry attempt requires hiring capital and skilled labour, both for the purpose of performing R&D and managerial activities. The cost function mirrors the incumbents' case and the normalised value of entry can be stated as:

$$v_t^e = \max_{\chi_t^e\in[0,1]}\left\{-(1+\zeta^e r_t^l)\omega_t\left(f^e + \frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) + \chi_t^e \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\frac{Y_{t+1}}{Y_t}\right)v_{t+1}(\phi_{t+1}^e)\right]\right\}$$

where χ_t^e is the desired entry probability, ζ^e denotes the share of factor rental costs that has to be paid in advance and borrowed at lending rate, f^e is the fixed cost of operation of potential entrants, a^e is the efficacy of R&D inputs in the case of entrants, and ϕ_{t+1}^e represents the expected relative quality level determined upon successful entry.

The first order condition of the potential entrants is given as follows:

$$0 = -(1+\zeta^e r_t^l)\frac{\omega_t}{a^e}\frac{1}{(1-\chi_t^e)^2} + \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\frac{Y_{t+1}}{Y_t}\right)v_{t+1}(\phi_{t+1}^e)\right] \tag{13}$$

The unbounded mass of potential entrants implies that whenever the expected value of entry is positive, more candidates attempt to enter the market, driving up the effective costs, and ensuring that the free entry condition holds:

$$v_t^e = 0 \tag{14}$$

[1] See the Web Appendix to Melitz and Redding [2014] or Bielecki [2017] for a proof. For empirical support see, e.g., Axtell [2001].

Following the observations of Acemoglu and Cao [2015] and Garcia-Macia et al. [2019], entrants enjoy a degree of entry advantage. To rule out limit pricing in equilibrium, it is assumed that entrants draw their quality from an appropriately upscaled quality distribution of incumbents, and the expected relative quality level upon entry is given by:

$$\mathrm{E}_t\left[\phi_{t+1}^e\right]=\frac{\sigma}{\sigma-1}$$

By denoting with M_t^e the mass of successful entrants, one can pin down the mass of effective potential entrants, which is then given by M_t^e / χ_t^e. Entry is constrained by the supply of skilled resources and is implicitly given by:

$$\frac{\left(K_t^s\right)^\alpha\left(Q_t N_t^s\right)^{1-\alpha}}{Q_t}=M_t f+\left(M_t-M_t^x\right)\left(\frac{1}{a}\frac{\chi_t}{1-\chi_t}\right)+\frac{M_t^e}{\chi_t^e}\left(f^e+\frac{1}{a^e}\frac{\chi_t^e}{1-\chi_t^e}\right) \tag{15}$$

where the left-hand side equals the effective supply of skilled input, which is (on the right-hand side) split between the operating cost of incumbents, R&D activities by non-exiting incumbents, and finally the operating cost and R&D activities of potential entrants.

The endogenous probability of an incumbent receiving a death shock depends on the exogenous, constant component, and the rate of entry of new establishments that potentially creatively destroy existing establishments:

$$\delta_t=1-\left(1-\delta^{exo}\right)\left(1-M_t^e\right) \tag{16}$$

The dynamics of establishment mass is then given by:

$$M_{t+1}=M_t-M_t^x-\delta_t\left(M_t-M_t^x\right)+M_t^e \tag{17}$$

Finally, the rate of change in the aggregate quality index depends on the intensity of the R&D activities performed by the incumbents and the mass of new entrants relative to active establishments. Since the quality levels are Pareto distributed, there exists a closed form expression for the rate of growth of the aggregate quality index:

$$\eta_t=\left(1-\chi_t+\chi_t\iota\right)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right) \tag{18}$$

Labour markets

The labour markets are assumed to be subject to the search and matching mechanism and the staggered real wages friction introduced in Gertler and Trigari [2009] and Gertler et al. [2008]. As in Christiano et al. [2011], the hiring and wage bargaining activities are relegated to employment agencies which then sell labour services to incumbents and entrants at a uniform wage.

The unskilled and skilled markets are assumed to be separated and they do not directly influence each other. Therefore, I only present the equations governing the behaviour of a representative labour market, as the mechanics of the skilled labour market is symmetrical, and I omit superscripts to ease the exposition.

The mass of unemployed workers is equal to:

$$u_t=1-n_t \tag{19}$$

The pool of unemployed is matched with the available vacancies according to the following matching function:

$$m_t=\sigma_m u_t^\psi v_t^{1-\psi} \tag{20}$$

where parameter σ_m describes the efficiency of the matching process and ψ is the elasticity of matches with respect to the mass of the unemployed.

Job finding probability p_t and job filling probability q_t can be obtained via the following transformation:

$$p_t = m_t / u_t \tag{21}$$

$$q_t = m_t / v_t \tag{22}$$

Employment agencies choose the hiring rate defined as:

$$x_t = m_t / n_t \tag{23}$$

And aggregate employment follows the law of motion:

$$n_{t+1} = \left(\rho + x_t\right) n_t \tag{24}$$

where $1-\rho$ is the exogenous, constant separation rate.

An individual employment agency supplies labour to producers at the common wage, but the wages paid to workers potentially differ between the agencies. Conditional on the offered wage, an employment agency chooses its hiring rate that maximises the net value of an additional hired worker, subject to convex costs with respect to its hiring rate:

$$J_t(j) = \max_{x_t(j)} \left\{ \tilde{w}_t - w_t(j) - \frac{\kappa}{2} x_t^2(j) + \left(\rho + x_t(j)\right) \mathrm{E}_t \left[\Lambda_{t,t+1} J_{t+1}(j) \right] \right\}$$

On the worker side, the values of being in the employed and unemployment states are given by the following formulas:

$$\mathcal{E}_t(j) = w_t(j) + \mathrm{E}_t \left[\Lambda_{t,t+1} \left[\rho \mathcal{E}_{t+1}(j) + \left(1-\rho\right) \mathcal{U}_{t+1} \right] \right]$$

$$\mathcal{U}_t = b_t + \mathrm{E}_t \left[\Lambda_{t,t+1} \left[p_t \mathcal{E}_{t+1} + \left(1 - p_t\right) \mathcal{U}_{t+1} \right] \right]$$

where the unemployed worker engages in undirected search, resulting in the expected value of being newly hired expressed as:

$$\mathcal{E}_t \approx \int \mathcal{E}_t\left(w_t\right) \mathrm{d}G\left(w_t\right)$$

where G denotes the cumulative distribution of wages and the above approximation is valid to the first order.

Accordingly, the surplus of a worker employed by agency j and the average surplus of newly hired workers equal:

$$H_t(j) = \mathcal{E}_t(j) - \mathcal{U}_t$$

$$H_t = \mathcal{E}_t - \mathcal{U}_t$$

And the surplus of being a new hire can be expressed as:

$$H_t(j) = w_t(j) - b_t + \mathrm{E}_t \left[\Lambda_{t,t+1} \left[\rho H_{t+1}(j) - p_t H_{t+1} \right] \right]$$

Wage bargaining follows the Nash bargaining procedure, although the parties involved realise that the wage is not renegotiated on a period-by-period basis. Moreover, new hires receive the wage prevailing at the employment agency. Once the agency receives a signal to renegotiate, the wage maximises the following Nash product:

$$w_t(r) = \mathrm{argmax} H_t(r)^{\psi} J_t(r)^{1-\psi}$$

subject to the staggered wage contract friction:

$$w_t(j) = \begin{cases} w_t(r) & \text{with probability } 1-\lambda \\ w_{t-1}(j) \cdot Q_t / Q_{t-1} & \text{with probability } \lambda \end{cases}$$

where in the case of being unable to renegotiate the wages are updated according to the rate of growth of the aggregate quality index. This assumption is similar to the inflation indexation often assumed in the Calvo schemes, popularized by **Christiano et al.** [**2005**], and ensures that the wage dispersion is a second-order phenomenon and can be omitted under first-order approximation, facilitating solution. The Calvo friction implies that a wage contract lasts for $1/(1-\lambda)$ periods on average and the average wage evolves as follows:

$$w_t = \lambda \frac{Q_t}{Q_{t-1}} w_{t-1} + (1-\lambda) w_t(r) \tag{25}$$

The solution of the Nash bargaining problem results in the conventional surplus sharing formula:

$$\psi J_t(j) = (1-\psi) H_t(j)$$

If the wages could be renegotiated each period, this flexible contract wage would be identical across all employment agencies and equal to:

$$w_t^f = \psi \left(\tilde{w}_t + \frac{\kappa}{2} x_t^2 + p_t \kappa x_t \right) + (1-\psi) b_t \tag{26}$$

In the case of staggered contracts the contract wage is given by:

$$\begin{aligned} \Delta_t w_t(r) = w_t^f &+ \psi \left(\frac{\kappa}{2} \left(x_t^2(r) - x_t^2 \right) + p_t \kappa \left(x_t(r) - x_t \right) \right) \\ &+ (1-\psi) p_t \mathrm{E}_t \left[\Lambda_{t,t+1} \lambda \Delta_{t+1} \left(w_{t+1} - w_{t+1}(r) \right) \right] \\ &+ \rho \lambda \mathrm{E}_t \left[\Lambda_{t,t+1} \Delta_{t+1} w_{t+1}(r) \right] \end{aligned} \tag{27}$$

where:

$$\Delta_t = \mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta \rho \lambda \right)^s \Lambda_{t,t+s} \frac{Q_{t+s}}{Q_t}$$

The above equation emphasises the presence of spill-overs of economy-wide wages on the bargaining wage. Intuitively, more intensive hiring by an agency requires higher bargained wages, which are also upwardly pressured by the future average wage.

Capital goods producers and the financial system

Perfectly competitive capital goods producers are also the owners of the capital stock which they rent to the establishments. They also borrow from the financial intermediary, at lending interest rate r_t^l, in order to finance investment in new capital. Therefore, they aim to maximise the expected discounted flow of their per-period profits, expressed as

$$\Pi_t^k = r_t^k K_t - I_t + L_{t+1}^k - \left(1 + r_t^l\right) L_t^k$$

where I_t is aggregate investment and L_t^k are loans from the financial intermediary. Physical capital accumulation is subject to the standard constraint:

$$K_{t+1} = I_t + (1-dp) K_t \tag{28}$$

The solution of the capital goods producers' problem yields the following equality between the lending rate and the capital rental rate net of depreciation:

$$r_t^l = r_t^k - dp \tag{29}$$

The financial intermediaries collect deposits from the households and lend them to two types of entities: intermediate goods producers and potential entrants, and capital producers. The profit of the intermediaries is given by:

$$\Pi_t^f = \left(1+r_t^l\right)L_t^k + \left(1+r_t^l\right)L_t^c - \left(1+r_t^d\right)d_t + d_{t+1} - L_{t+1}^k - L_{t+1}^c$$

where L_t^c denotes loans to establishments to finance their working capital requirement, and subject to the loanable funds constraint:

$$L_{t+1}^k + L_{t+1}^c \leq d_{t+1}$$

The financial intermediaries are owned by the households and discount the future in the same manner. Here I make the assumption that the financial intermediaries enjoy a degree of market power that drives a wedge between the deposit and lending interest rates, such that:

$$r_t^l = r_t^d + sp_t \tag{30}$$

where sp_t is the spread between the interest rates. When taken literally the variation in the spread would imply that the market power of banks is changing over time, but it should be interpreted as a reduced-form way to capture the frictions in the financial markets. The spread is assumed to evolve according to the following AR (1) process:

$$\log sp_t = \left(1-\rho_{sp}\right)\log sp_{ss} + \rho_{sp}\log sp_{t-1} + \varepsilon_{sp,t} \tag{31}$$

Market clearing

The markets for factors of production clear:

$$N_t^p = (1-s)n_t^u \quad \text{and} \quad N_t^s = sn_t^s \tag{32}$$

$$K_t = K_t^p + K_t^s \tag{33}$$

As the households are subject to within-family risk sharing and behave in a Ricardian manner, there is no need to explicitly model a fiscal authority, which in the background collects lump-sum taxes and provides unemployment benefits, as well as various subsidies discussed later in the paper. What needs to be ensured, however, is that the final goods output is spent on consumption, investment and covering hiring costs:

$$Y_t = C_t + I_t + \kappa^u \left(x_t^u\right)^2 N_t^p + \kappa^s \left(x_t^s\right)^2 N_t^s \tag{34}$$

Finally, the process for aggregate productivity is assumed to follow the standard AR (1) form:

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \tag{35}$$

Data and results

Data, calibration and estimation

The data used in the paper come from a couple of sources. The data on establishment dynamics comes from the US Bureau of Labor Statistics (BLS) Business Employment Dynamics (BDM) database. The BDM, based on the Quarterly Census on Employment and Wages (QCEW), records changes in the employment level of more than 98% of economic entities in the United States, which makes it possible to track the cyclical behaviour of establishments. Unfortunately, the data series is relatively short, starting in the third quarter of 1992. The data on GDP and its components come from the US Bureau of Economic Analyses (BEA). The labour market statistics come predominately from the BLS. To construct the data on vacancies, the data from the JOLTS survey, available from December 2000, were spliced with the Composite Help Wanted Index provided by **Barnichon** [**2010**]. Following **Christiano et al.** [**2014**], the series for the interest spread was chosen as Moody's Seasoned BAA Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity, provided by the Federal Reserve Bank of St. Louis, starting in April 1953. The data on R&D spending were taken from the BEA and the National Science Foundation (NSF). The NSF also provides data on the full-time equivalent number of employees performing R&D, although the series ends in 2008.

The model is calibrated to replicate the key features of the US economy. The parameters that influence the steady state of the economy are calibrated to reflect the long-run averages in the US data and are summarised in Table 1. The average quarterly spread was calculated directly from the corresponding data series. The degree of pre-financing was taken from **Christiano et al.** [**2010**]. The values of the parameters governing the behaviour of the labour markets were taken from previous literature. The differentiated separation rates for unskilled and skilled workers are taken from **Cairo and Cajner** [**2018**] and adapted to the quarterly model set-up. The adjustment cost parameters were chosen to match the average job finding probability in the United States, which **Shimer** [**2005**] reports to be equal to 0.45 and **Cairo and Cajner** [**2018**] document that the job finding probabilities differ only slightly among the workers' education groups. As in **Shimer** [**2005**] the unemployment benefits are assumed to be equal to 40% of the steady state wage. Following **Gertler and Trigari** [**2009**], the elasticity of matches to unemployment is set to 0.5 and the condition that the bargaining power parameters correspond to matching elasticities is imposed. Finally, the matching efficiency parameter is set to match the observed average vacancy to an unemployment ratio of 0.61.

Both the capital share of income and the quarterly depreciation rate are set to values ubiquitous in the business cycle literature. Note that since the firms in the model generate positive profits, the labour share of income is slightly lower than $1-\alpha$. The discount factor, which in the calibration process depends on the value of elasticity of intertemporal substitution, is chosen so that the average annual net deposit interest rate is equal to 4.75%. This, together with the assumed average spread, implies that the lending rate, equal to the rate of return on capital, is 6.65%, a value consistent with literature, see e.g. **Nishiyama and Smetters** [**2007**].The share of skilled workers is picked to be in the middle of the plausible range of values proposed by **Acemoglu et al.** [**2018**] and corresponds to the value used by **Bielecki** [**2017**] and adjusted to account for the presence of unemployment in the model. Finally, the set of parameters governing the establishment dynamics is calibrated to match the specific moments reported in Table 2. The fixed cost and R&D efficacy of the entrants are assumed to be exactly the same as for the incumbents, with discerning notation only introduced to facilitate the application of targeted subsidy schemes. Finally, the set of six parameters governing the establishment dynamics is calibrated to match the specific six moments reported in Table 2, which are all matched almost exactly.

Table 1. Calibrated parameters affecting the steady state

Parameter	Description	Value	Justification
sp_{ss}	Average quarterly spread	0.0047	Annualised spread = 1.9%
ζ, ζ^e	Working capital share	0.75	**Christiano et al.** [**2010**]
ρ^u	Unskilled retention rate	0.9197	**Cairo and Cajner** [**2018**]
ρ^s	Skilled retention rate	0.9703	**Cairo and Cajner** [**2018**]
κ^u	Unskilled hiring cost	2	Unskilled job finding probability
κ^s	Skilled hiring cost	15.8	Skilled job finding probability
b^u	Unskilled unemp. benefit	0.14	40% of steady state unskilled wage
b^s	Skilled unemp. benefit	0.41	40% of steady state skilled wage
ψ	Elasticity of matches	0.5	**Gertler and Trigari** [**2009**]
σ_m	Matching efficiency	1.7	Average tightness = 0.61
α	Capital share of income	0.3	Standard

Parameter	Description	Value	Justification
dp	Capital depreciation rate	0.025	Standard
β	Discount factor	0.9996	Annual net interest rate of 4.75%
s	Share of skilled workers	0.1039	**Bielecki** [**2017**]
ι	Innovative step size	1.019	Annual pc. GDP growth
δ^{exo}	Exog. exit shock prob.	0.016	Exit rate
a, a^e	R&D efficiency	7.34	Expansions = contractions
f, f^e	Operating fixed cost	0.84	Share of R&D in GDP
θ	Inverse of IES	2.31	Share of investment in GDP
σ	Elasticity of substitution	5.23	Share of R&D employment

Source: Own calculations.

Table 2. Long-run moments: comparison of model and data

Description	Model	Data	Source
Annual pc. GDP growth	2.01%	2.01%	BEA, 1948q1-2019q4
Exit rate	3.02%	3.02%	BDM, 1992q3-2019q4
Relative share of expanding estabs.	1.01	1.01	BDM, 1992q3-2019q4
Share of R&D in GDP	2.45%	2.49%	BEA, 1948q1-2019q4
Share of investment in GDP	17.59%	17.56%	BEA, 1948q1-2019q4
Share of R&D employment	0.96%	0.98%	NSF & CBP, 1964–2008

Source: Own calculations.

Table 3. Prior distributions of parameters

Parameter	Description	Distribution shape	Mean	Std. dev.
λ	Calvo wage contract prob.	Uniform [0, 1]	0.5	0.289
ρ_z	Autocorr. of prod. process	Uniform [0, 1]	0.5	0.289
σ_z	Std. dev. of prod. shock	Inverse Gamma	0.01	∞
ρ_{sp}	Autocorr. of spread process	Uniform [0, 1]	0.5	0.289
σ_{sp}	Std. dev. of spread shock	Inverse Gamma	0.1	∞

Source: Own calculations.

The values of the parameters that do not affect the steady state but govern the cyclical behaviour of the model were obtained via estimation. The prior distributions were chosen to be relatively uninformative, and in particular the prior distribution for the renegotiation frequency parameter was set to uniform on the unit interval. Table 3 contains full information on the priors used.

The estimation makes use of two observable data series. The first one is the growth rate of real gross domestic product divided by the labour force, observed from the second quarter of 1948 to the fourth quarter of 2019. An advantage of the model with explicitly modelled long-run growth is that there is no need to detrend the data and valuable information is retained. The second is the demeaned spread between BAA and long-term government bonds. The model was estimated using standard Bayesian procedures with the help of Dynare 4.5.7 and the results were generated using two random walk Metropolis-Hastings chains with 200,000 draws, each with an acceptance ratio of 0.24.

Table 4 presents the estimation results. The data were clearly informative about the estimated parameters, as the posterior and prior means differ and the highest posterior density (HPD) intervals are relatively tight.

The most interesting parameter is the one regulating the contract renegotiation probability, and its value implies that wage contracts last for six quarters on average. This value is slightly higher than assumed by **Gertler** and Trigari [2009] in their calibrated model, where they consider average duration of nine and 12 months, and also higher than estimated by Gertler et al. [2008], where contracts last for three-and-a-half quarters[2]. However, assuming this value of the parameter yields excellent performance in the case of labour market variables.

Table 4. Prior and posterior means of parameters affecting cyclical behaviour

Parameter	Description	Prior mean	Post. mean	90% HPD interval
λ	Calvo wage contract prob.	0.5	0.861	[0.766, 0.950]
ρ_z	Autocorr. of prod. process	0.5	0.941	[0.896, 0.991]
σ_z	Std. dev. of prod. shock	0.01	0.011	[0.011, 0.013]
ρ_{sp}	Autocorr. of spread process	0.5	0.935	[0.895, 0.968]
σ_{sp}	Std. dev. of spread shock	0.1	0.162	[0.151, 0.174]

Source: Own calculations.

Model performance and impulse response functions

The data moments were generated on the sample 1948q1-2019q4, with the exception of vacancies and tightness, available from the first quarter of 1951, and establishment dynamics, available from the third quarter of 1992. The variables trending with population growth, such as GDP and the number of establishments, were divided by the Civilian Labour Force and subsequently detrended with the Hodrick-Prescott filter.

Table 5 presents the comparison of the Hodrick-Prescott filtered moments between the model and data. The upper section of the table is concerned with output and its components, as well as R&D expenditures. The model fits the data very well for output and its components, and only fails to account for the much weaker correlation of R&D expenditures with output.

The middle section of the table focuses on variables pertaining to the operations of the labour market. The model wages are more strongly correlated with output and have higher autocorrelation than in the data, and the model hours are not as volatile as in the data. However, the model is very successful in matching the cyclical behaviour of unemployment, vacancies and tightness, achieving a nearly perfect fit.

Table 5. Business cycle moments: comparison of model and data

	Standard deviation		Correlation with *Y*		Autocorrelation	
Variable	Data	Model	Data	Model	Data	Model
Output	1.54	1.54	1.00	1.00	0.82	0.82
Consumption	0.87	0.74	0.78	0.90	0.82	0.75
Investment	4.54	5.34	0.76	0.98	0.87	0.89
R&D	2.39	2.08	0.31	0.94	0.89	0.92
Wages	0.94	0.81	0.17	0.41	0.69	0.96
Hours	1.36	0.65	0.86	0.92	0.90	0.90
Unemployment	12.5	13.3	−0.79	−0.82	0.89	0.90
Vacancies	13.6	14.9	0.87	0.92	0.91	0.88
Tightness	25.5	27.2	0.85	0.89	0.91	0.92
Establishments	0.57	0.22	0.69	−0.38	0.95	0.90
Expansions	2.74	0.45	0.79	0.78	0.73	0.94
Contractions	2.42	0.58	−0.07	−0.93	0.67	0.90
Net Entry	0.21	0.10	0.39	0.18	0.86	0.48

Source: Own calculations.

2 Note, however, that Gertler et al. [2008] impose a relatively tight prior on this parameter.

The final section presents the moments related to the establishment dynamics. Although the fit is a bit worse than in the case of previously discussed variables, most of the model moments remain close to their data counterparts, with the exception that the model predicts much smaller volatility of establishment dynamics. The model also predicts that the establishment mass is slightly negatively correlated with output, even though the correlation of net entry with output is almost exactly the same as in the data. A brief look at the impulse response functions in Figure 1 reveals that this result is most likely driven by a small and short-lived decrease in the mass of establishments immediately after the shock hits, and for the subsequent periods the mass of active establishments moves in tandem with output. In the case of interest rate spread shocks, Figure 2 shows that output and establishment mass co-move.

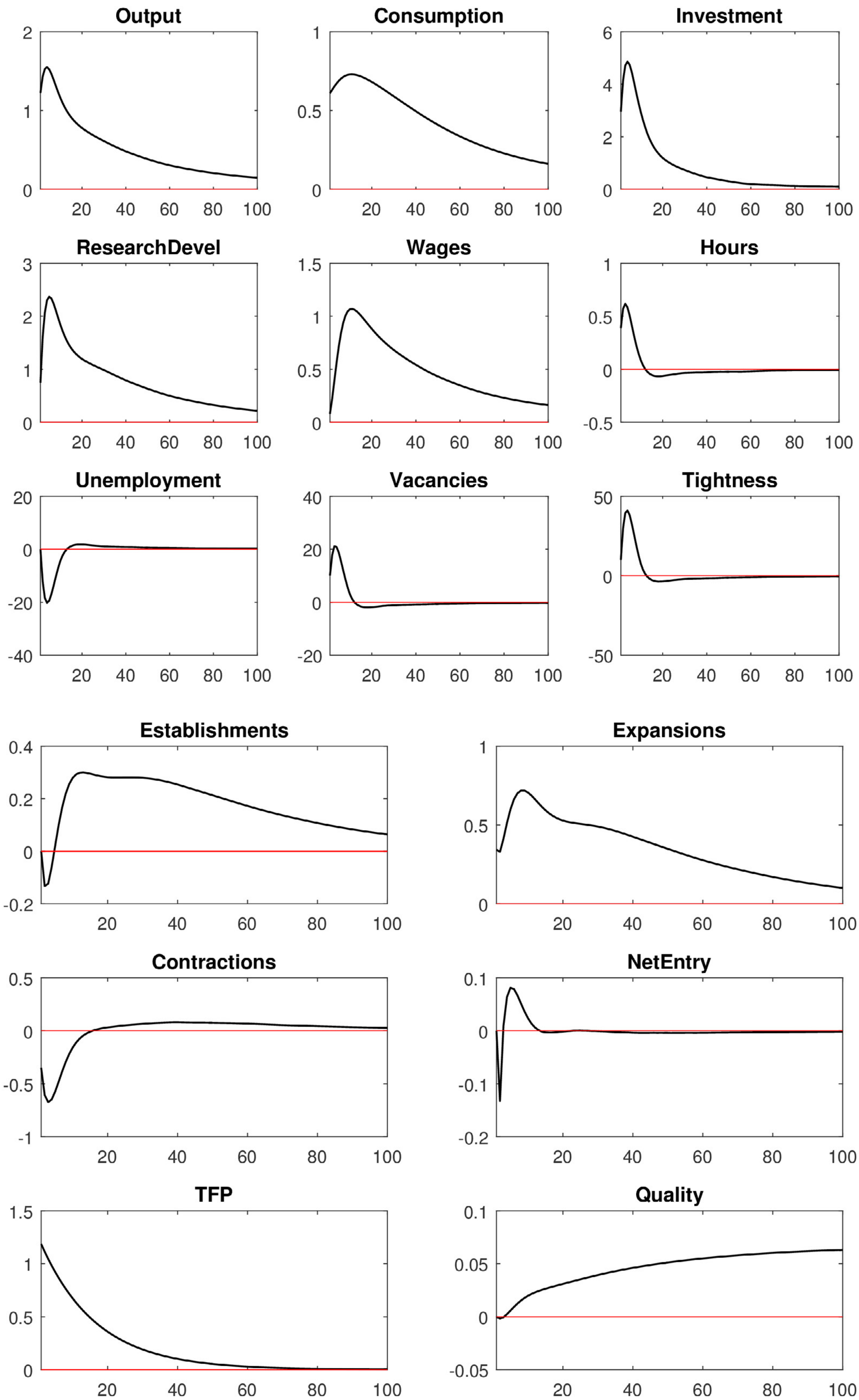
Figure 1 displays the impulse response functions to a standard deviation productivity shock. An increase in productivity raises output both directly and indirectly, through higher investment and hiring rates, which in turn cause an increase in the physical capital stock and hours worked. The response of output to the shock is highly persistent, both due to labour market frictions and the endogenous quality component, which permanently shifts output upwards. Expenditures on R&D are also procyclical and persistent.

Staggered wage contract friction prevents wages from responding strongly on the impact of the shock, as a large percentage of employment agencies is not allowed to renegotiate wages. Over time, wage renegotiations take place, and the response of the wages exhibits a hump-shaped pattern, reaching a peak at around three years after the initial shock. An increased productivity of labour induces employment agencies to post vacancies, increasing labour market tightness.

A positive productivity shock incentivises incumbents to increase their R&D expenditures and consequently success probability, and as a consequence the mass of expanding establishments increases, while the mass of contracting establishments decreases. Strongly elevated demand for skilled labour results in a temporary decline in net entry rates on shock impact. However, as more skilled employees become available, net entry turns positive and translates into an increase in the active establishments mass. The rate of growth for the aggregate quality index is higher than along the balanced growth path, due to both higher R&D intensity and entry rates. This faster pace of growth is at first maintained by both higher skilled employment and bigger capital stock, although after around four years employment returns to its balanced growth path level, leading to a decrease in the rate of quality growth. Nevertheless, more abundant physical capital allows the economy to continue growing faster, and eventually the level of quality relative to the balanced growth path trend flattens out and stabilises at a level around 7% higher than it would be if the shock never happened.

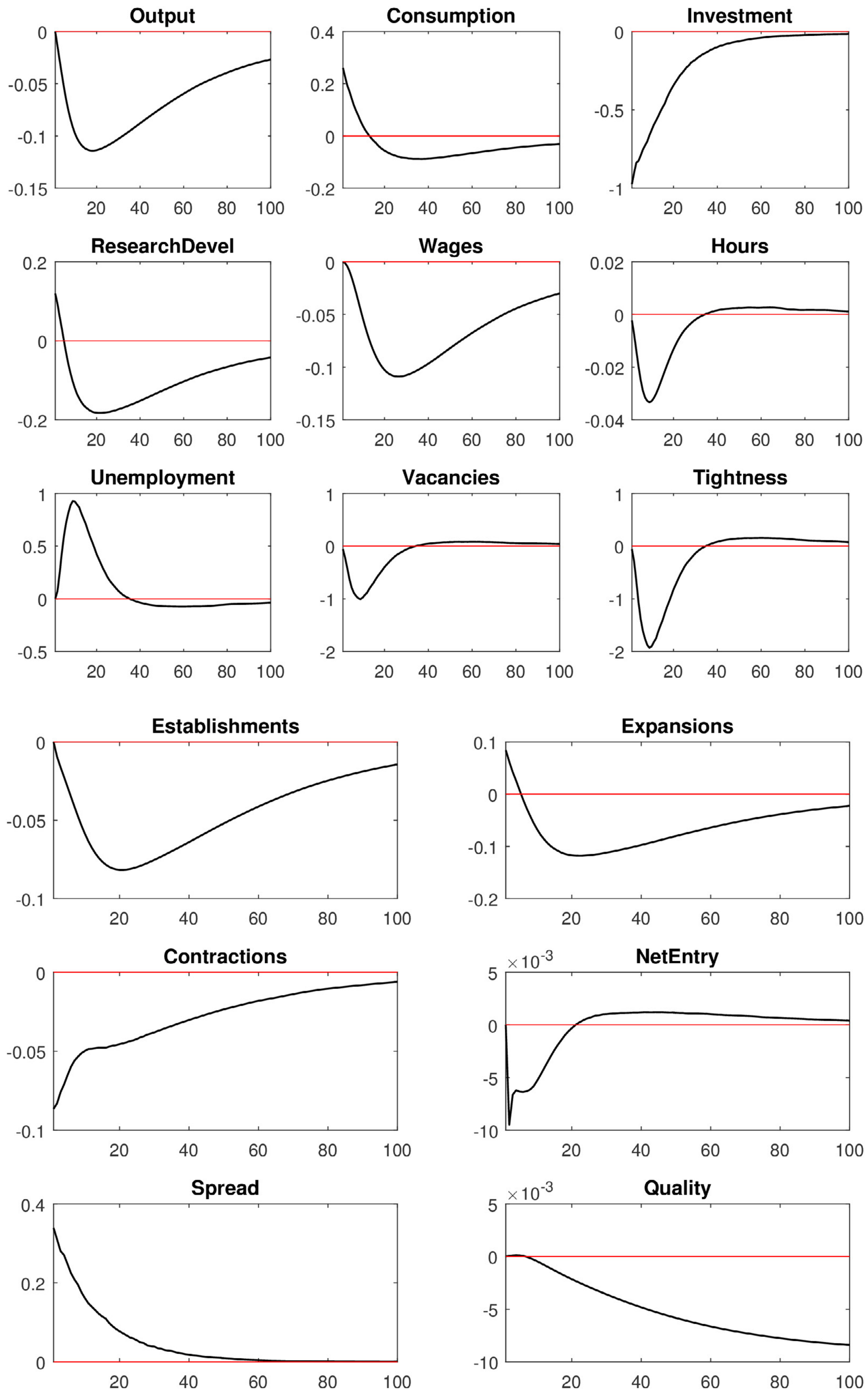
Figure 2 displays the impulse response functions to a standard deviation interest spread shock. Broadly speaking, an increase in the wedge between the deposit and lending rates generates effects opposite to the positive productivity shock, and their quantitative size is of a smaller order of magnitude. Investment in physical capital decreases on shock impact, as it is now more costly to produce new units of physical capital, and consumption rises in response to lower deposit interest rates. Expenditures on R&D initially increase, as incumbents face a lower risk of being creatively destroyed due to decreased entry, but after about a year they drop below the balanced growth path trend as the recession deepens. Both the hours worked and the wages decrease in a hump-shaped pattern, while unemployment increases. Creating new vacancies is discouraged, and as a result the labour market becomes less tight. Increased costs of lending deter entry, which remains depressed for about five years after the initial shock, which also causes a decrease in the mass of establishments. The aggregate quality level remains near its trend level for around two years following the shock, as expansions and net entry move in opposite directions. After that period both depressed entry and incumbents' R&D intensity translate into a downward deviation of the quality level from trend, which is eventually lowered by about 0.85% relative to its trend path. Thus the financial shocks, compared to productivity shocks, create similar – though smaller in magnitude – shifts in the balanced growth path of the economy.

Figure 1. Impulse response functions to standard deviation productivity shock



Source: Own calculations.

Figure 2. Impulse response functions to standard deviation interest rate spread shock



Source: Own calculations.

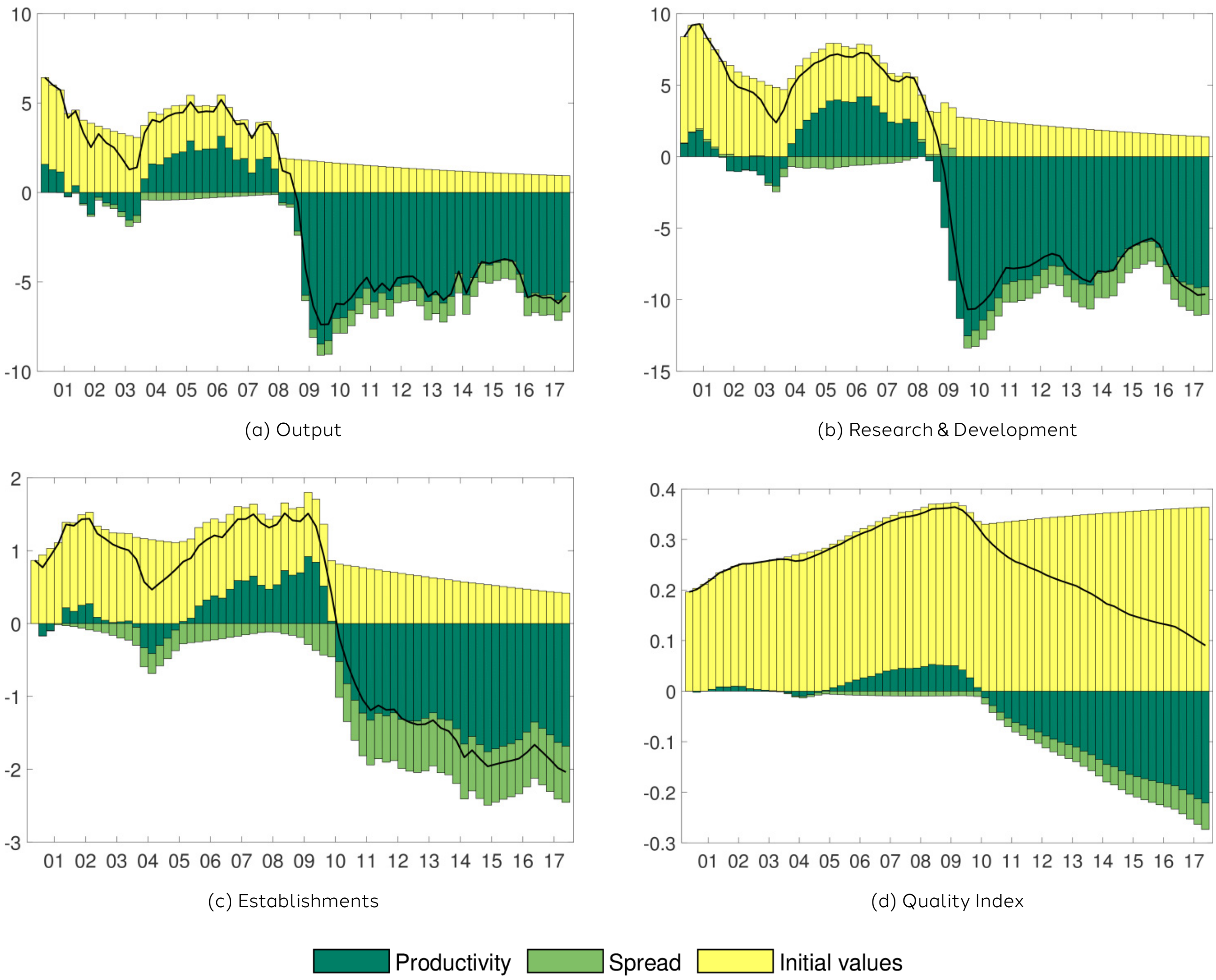
Long shadows of financial shocks

The experience of the Great Recession

The model features mechanisms through which temporary shocks translate into permanent shifts to the balanced growth path of the economy. Therefore it is an attractive laboratory to study the experience of the Great Recession.

The Great Recession has been associated with the largest output drop in the post-war economic history of the United States, which until late 2019 remained around 10% below its pre-recession trend. A similar behaviour was observed for R&D expenditures, although the drop was even deeper than for output. Increased establishment exits and depressed entry have resulted in fewer active establishments.

Figure 3. Shock decomposition of key macroeconomic variables



Source: Own calculations.

Figure 3 presents the shock decomposition of key macroeconomic variables from the first quarter of 2000 to the second quarter of 2017. The financial shocks, modelled as increases in the spread between the deposit and lending interest rates, account for a nontrivial fraction of the deviation of the variables from their trend. In particular, about a third of the total decline in the establishment mass is attributed to increased spreads, as they are especially harmful to entrants. Depressed entry rates and R&D expenditures result in a continuing fall of the aggregate quality index. This has profound implications as, while the physical capital stock and employment levels can in principle return to their balanced growth path levels, a decline in the aggregate quality is of a more permanent nature and essentially pushes the economy to a balanced growth path below the pre-crisis one.

Policy implications

Since temporary shocks can exert level effects on the balanced growth path of the economy, this implies that business cycle fluctuations are associated with additional welfare costs compared to the models where growth results from exogenous processes.

To quantify the welfare comparisons across different states of the world, second-order approximation of the model equations is employed and the consumption equivalents are calculated. For the utility function assumed in this paper, the equivalent-adjusted welfare is given by:

$$W_0(eq) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left((1+eq)c_t\right)^{1-\theta}}{1-\theta} = (1+eq)^{1-\theta}\, \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta}$$

The consumption equivalent across two different worlds can be then computed as follows:

$$eq_{a,b} = \left(U_0^b / U_0^a\right)^{\frac{1}{1-\theta}} - 1$$

where U_0^a and U_0^b denote expected lifetime utilities in worlds *a* and *b* respectively. Then $eq_{a,b}$ has the interpretation of what proportion of consumption an agent living in world *a* would we willing to forfeit in order to "move" to world *b*.

Table 6 presents the comparison of the expected lifetime utilities in five distinct worlds. In the non-stochastic world, the economy is not subject to shocks and always remains on its balanced growth path. In the two stochastic worlds, the economy is affected by shocks but in the first of them growth is exogenous and the aggregate quality index increases at a constant rate. As a consequence, any welfare losses result from the volatility around the trend and are estimated to be quite low, in accordance with existing literature. The second stochastic world is identical to the model economy. Here welfare losses are significantly larger and stem from the fact that both shocks result in the level shifts of the consumption paths. Finally, the lower section of Table 6 is concerned with the relative importance of two shocks for welfare. It turns out that the spread shocks account for about a third of the total welfare costs.

Table 6. Welfare cost of business cycles

State of the world	Welfare	Consumption equivalent
Nonstochastic (BGP)	–172.84	
Stochastic with exogenous growth	–172.97	0.05%
Stochastic with endogenous growth	–191.13	8.04%
Endogenous growth without spread shocks	–178.85	2.66%
Endogenous growth without prod. shocks	–185.52	5.60%

Source: Own calculations.

The presence of significant welfare costs of business cycles poses questions about whether economic policy can alleviate some of them. The subsequent analysis examines the macroeconomic and welfare effects of applying several subsidy schemes. Those schemes fall into two groups: static and countercyclical subsidies. All subsidy schemes are financed via lump-sum taxation.

Static subsidies are designed to act as if a parameter in question was changed by 10%. The direction of change is always in the direction favoured by the subsidised agent, e.g. a lowering of operation costs or increasing

R&D efficiency. Table 7 documents the results of applying subsidies to the operation cost of incumbents and prospective entrants, their R&D expenditures, reducing their working capital requirement, and reducing the steady-state spread. The first result column displays the average growth rate in stochastic equilibrium. The next two present the extent of change in the aggregate quality index in response to a standard-deviation productivity shock, over the horizon of 20 and 100 quarters respectively, while the following two columns do the same for the spread shock. The next column reports the expected lifetime utility measure, and the following one provides the average unemployment rate in stochastic equilibrium. For ease of interpretation, the last column presents the opposite number to the consumption equivalent, so that the positive value of the statistic indicates the welfare gain. As a rule of thumb, *ceteris paribus*, households prefer if the aggregate growth rate is higher, and volatility (understood as the extent of the reaction of aggregate quality in response to the shock) and the unemployment rate are lower.

In agreement with the endogenous growth literature, I find that subsidising the R&D expenditures of incumbent establishments is strongly welfare improving as the average growth rate increases while volatility decreases at the expense of a slightly higher unemployment rate. Welfare gains are also associated with lowering barriers to entry, either through lowering the fixed costs of prospective entrants or subsidising their R&D activities. Contrary to some previous literature, e.g. **Acemoglu et al. [2018]**, subsidising incumbents' operating costs is found to be welfare improving. This discrepancy stems from the fact that although the subsidised economy exhibits a lower rate of growth and higher volatility, those effects are dwarfed by gains from decreased churning in the labour market, the full extent of which becomes apparent only in the stochastic setting.

The lower section of the table presents the effects of subsidies that aim to reduce frictions in the financial markets. Subsidising the working capital costs of incumbents lowers slightly the volatility of the economy and generates a small welfare gain, while subsidies to the working capital of entrants do not have a significant welfare effect. Finally, subsidising all borrowers in a manner that acts as if the average spread was lower, decreases both volatility and the average unemployment rate, resulting in welfare gains.

Table 7. Effects of static subsidies

	γ	ΔQ_{20}^{Z}	ΔQ_{100}^{Z}	ΔQ_{20}^{sp}	ΔQ_{100}^{sp}	U	u	$-eq$
Baseline	2.04	3.17	6.32	0.21	0.82	−191.13	5.65	
f	2.02	3.33	6.63	0.24	0.87	−187.07	5.53	1.64%
f^e	2.04	3.18	6.36	0.22	0.82	−190.91	5.65	0.09%
a	2.10	3.08	6.12	0.22	0.80	−186.97	5.67	1.68%
a^e	2.05	3.17	6.34	0.22	0.82	−191.05	5.65	0.03%
ζ	2.05	3.16	6.32	0.21	0.82	−190.92	5.65	0.09%
ζ^e	2.05	3.17	6.32	0.21	0.82	−191.15	5.65	−0.01%
sp_{ss}	2.05	3.16	6.31	0.19	0.74	−189.28	5.63	0.75%

Source: Own calculations.

Table 8 presents the welfare effects of countercyclical subsidies. As they by construction do not impact significantly the economy's average growth rate or unemployment rate, those variables are not displayed. Two variants of subsidies are considered: in the first, if output is observed at a level 1% lower than trend, the subsidy increases by 0.5%. Conversely, in times of boom, the subsidy becomes a tax. In the second variant, the subsidy increases by 5% if the spread is 1 percentage point higher than on average.

The qualitative effects of the two subsidy variants are almost identical. Intuitively, subsidy schemes that lower volatility bring welfare gains. The biggest welfare gains are associated with subsidising the operating costs of active establishments, which lends support to policies aimed at supporting existing firms during recessions. On the other hand, countercyclical subsidies to incumbents' R&D activities are welfare deteriorating, as, by redirecting limited resources towards incumbents, it exacerbates the difficulties entrants face during downturns. Finally, subsidies to entrants carry small positive welfare gains, while subsidies to working capital have almost no impact on volatility and welfare.

Table 8. Effects of countercyclical subsidies

	ΔQ_{20}^{Z}	ΔQ_{100}^{Z}	ΔQ_{20}^{sp}	ΔQ_{100}^{sp}	U	$-eq$
0.5% subsidy if output is 1% below trend						
Baseline	3.17	6.32	0.21	0.82	−191.13	
f	3.11	6.16	0.21	0.79	−187.00	1.67%
f^e	3.16	6.31	0.21	0.82	−190.99	0.06%
a	3.30	6.60	0.22	0.86	−195.82	−1.88%
a^e	3.17	6.32	0.21	0.82	−191.08	0.02%
ζ	3.17	6.32	0.21	0.82	−191.17	−0.01%
ζ^e	3.17	6.32	0.21	0.82	−191.13	0.00%
5% subsidy if spread is 1 percentage point above average						
Baseline	3.17	6.32	0.21	0.82	−191.13	
f	3.17	6.32	0.20	0.79	−189.28	0.75%
f^e	3.17	6.32	0.21	0.82	−191.07	0.02%
a	3.17	6.32	0.26	0.88	−193.20	−0.83%
a^e	3.17	6.32	0.21	0.82	−191.11	0.01%
ζ	3.17	6.32	0.21	0.82	−191.15	−0.00%
ζ^e	3.17	6.32	0.21	0.82	−191.13	0.00%

Source: Own calculations.

Conclusions

The Great Recession has resulted in a seemingly permanent level shift in many macroeconomic variables. This paper has presented an endogenous growth model where monopolistically competitive, heterogeneous establishments choose the level of R&D expenditures. The model economy is also subject to the search and matching friction in the labour market, as well as financial friction modelled as a reduced-form shock to the spread between the deposit and lending interest rates. This set-up generates volatile and procyclical R&D expenditure patterns and is consistent with the business cycle dynamics of GDP and its components, labour market variables, as well as establishment dynamics.

Both productivity and financial shocks affect the endogenous growth rate of the economy, resulting in level shifts in the balanced growth path. This significantly increases the estimate of the welfare costs of business cycles. As a consequence, economic policy can play an important role in alleviating the consequences of those shocks.

In the case of static subsidies, subsidising R&D expenditures, as well as lowering barriers to entry, were found to be welfare improving, in line with endogenous growth literature. At odds with this literature, static subsidies to incumbents' operating costs were also found to be welfare improving. This result stems from taking into account the effects of business cycle fluctuations in an economy with frictional labour and financial markets.

In the case of countercyclical subsidies, subsidising the R&D expenditures of active establishments is welfare deteriorating, as it redirects precious resources from more efficient uses. On the other hand, subsidising incumbents' operating costs is welfare enhancing, regardless of whether the economy is hit by a productivity or financial shock. This result supports implementing policies that aim to reduce the exits of active establishments during recessions.

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