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Endogenous and Exogenous Components of Economic Growth

Summary: The aim of the article is to explain the role of the main factors determining the level of economic growth in both the short and long term. The analysis concerns an open economy that is initially balanced inside and outside. It is assumed that capital investments are in balance with investments in innovation and that investments in new capital are in balance with investment replacing old capital. Unlike endogenous factors, which impact economic growth in a way similar – though not identical – to that described in the neoclassical model, exogenous factors to a large extent bear the features of multiplier interventions, which can be described with the help of a deeply modified IS-LM-BP model. In addition to long-term economic inertia and short-term multiplier impacts, there is also an immediate stimulation that needs to be considered in the model.

In the article, a fundamental model is presented that, on the demand side, is based on a debt function developed according to the author's own idea. The model also relies on a consumption function that to an extent is inspired by the rational expectations theory. As far as the supply side is concerned, the model is based on cost, revenue and profit functions inspired by R. Vernon's theory of product life cycles. These functions are synchronized and combined into a single model, which also incorporates the formally modified IS-LM-BP model.

The main conclusion from the analysis is that the contemporary economy is a heterogeneous entity. At the same time, it is market driven in the sense that it is possible not to differentiate among the different kinds of investments in the model. But the contemporary economy is also subject to impacts that are not seen as market driven under the neoclassical view.

Keywords: endogenous, exogenous, growth, economy, intervention

JEL classification codes: E12, E13, E17, E63, F41, F43, F47, H12, O11, O41

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Introduction

That economic growth depends on the previous investment level and consumption is a well-established fact and most of the theories on economic growth take into account these two factors, though in different ways and scope. Also, state and central bank interventions have been examined many times, and the same goes for international trade and international capital movements. This paper, however, offers a proposal for a new synthetic theory on this problem.

The basic assumption of the theoretical model, on the one hand, is that the present growth level of an economy is driven by the psychological attitudes of consumers, which are considered here to be a universal mathematical feature. On the other hand, the present growth level of an economy is assumed to be driven by the product life cycles of previous investments. The state, the central bank and their economic policy, as well as the impact of the balance of payments, are also included in the theory. In the section below, the general concept of the model is presented. The next section focuses on a mathematical analysis of the model. In the subsequent section, the state, the interventions and the balance of payments are included in the model mathematically. In the final section, a summary of the model is presented together with its long-term growth interpretation.

The model

First, we assume that the economy is about purchases. This means that home services and other free gifts are not part of the economy the way it is summarized in the GDP. This economy is about exchanging goods, and its primary feature is the exchange of one good for another. The basic good that can be exchanged into any other good is money (the *numéraire*). The basic decision-making market entity is the consumer, who exchanges his income for other goods. Consumers are also investors, either directly or through financial markets, as well as final owners of capital. The state is an exogenous entity.

Consumers generally have two feelings during such exchanges. One is the sorrow for having to part with the money; the other is being happy with acquiring the purchased goods. These two feelings, if described mathematically as average consumer behavior, are not governed by the same functions. The first of these functions, about parting with money, can be called the spending-debt impulse function; the second, about acquiring the goods – the consumption impulse function.

In order to set the spending-debt impulse function, later on simply called the debt impulse function, we will first engage in the following analysis. We assume that economic entities, indicated here as consumers¹, can be divided into three groups²:

¹ That also includes households. We use the term consumer as a somewhat more general entity here.

² Entities that do not possess income or capital do not participate in the economy in a substantial way.

- Class A entities who possess both income and capital;
 - Class B entities who do not possess income but possess capital;
 - Class C entities who possess income but do not possess capital;
- Class C entities are confident because of the feeling of security associated with a stable income. They look for credit that can be delivered to them by class B entities. These latter entities, because they do not have the income (in relative terms), instead of spending their accumulated financial capital, prefer to part with it for interest (that must include the risk involved). In this way “formal”³ debt appears through financial markets.

However, there also exists “non-formal” debt. This is the case with class A entities who possess both income and capital in a balance. When spending their capital these entities borrow from themselves. Following the expenditure of an amount of money and having large demand for the reserve money (which is naturally present for those entities that are able to accumulate the capital), they tend to recover the spent reserve of money on future income. This is “non-formal” debt because it is not expressed by any kind of market contract⁴. We assume here that both these kinds of debt are of the same binding strength in the economic system on average and we can term them together as “absolute” debt. Thus we may generalize that any expenditure is followed by time shifted debt⁵. In macroeconomic terms, the whole yearly yield which is spent (the GDP) generates its “absolute” debt. This approach is justified because, just as we can divide any income and expenditure into bits of income and expenditure, we can also aggregate bits of income and expenditure into total income and expenditure in a given period of time, if this time is not indefinitely long.

The following graph (Figure 1.2) depicts this spending-debt behavior.

When spending their income *ad hoc* over time, consumer entities in the economy create from a moment of time τ an “absolute” debt, some of which is psychological “non-formal” debt and some is the “formal” debt usually delivered through financial markets⁶. The surface between the curve and the time axis is the same above the axis and below it, which means that the “absolute” debt is equal to expenditure. The yields *ad hoc* overlap each other, and the same is true of the yearly yields and the entire yearly yields (GDP). In this way, the state of the debt in the economy at a given time will be represented by all parts of the curve, if the curve represents the average propensity to

³ That is contractual.

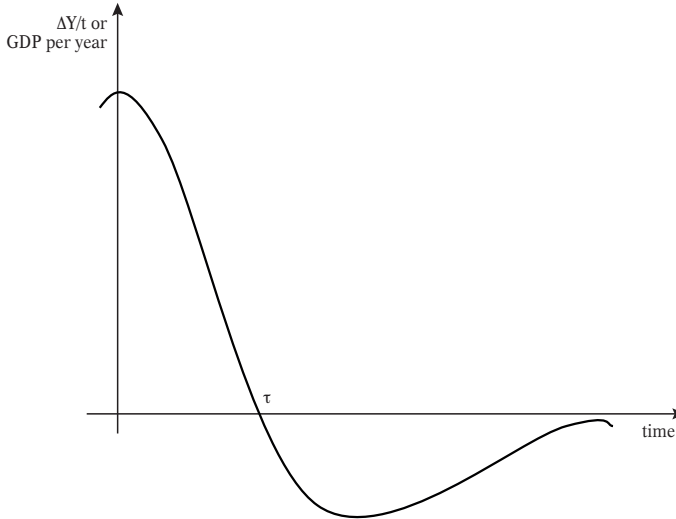
⁴ We can see this as a kind of “mental accounting”. The fact that some behaviors are driven by mental accounting has been reported and studied by researchers including Choi, Laibson & Madrian [5], Thaler [12], [13], [14].

⁵ We can see this as an induction made on the general observation that people and particularly wealthy people tend to recover their spent reserves on future income and this is also true with their income *ad hoc* and more generally with any possessed money.

⁶ It is, however, imaginable that the “formal” debt becomes greater than the “absolute” debt in some crisis formations and that the “absolute” debt is mostly represented by the “formal” debt.

spend or accumulate generated by all past yields. This will also be the case if the present is in τ .

Figure 1.2
The spending-debt impulse function



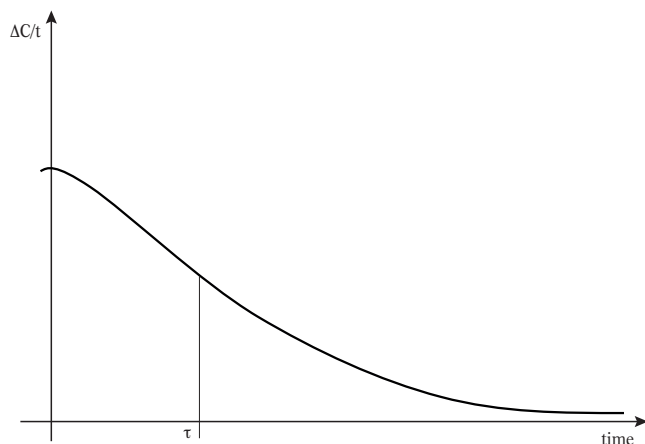
The consumption impulse function will then represent the second mentioned feeling, which is the greed for goods generated by an income *ad hoc* in the same way as the above debt impulse function represents the attitude to spending or accumulating money – Figure 2.2.

We assume that the consumption impulse function is a normal distribution function spread around an income *ad hoc* and that it also represents the aggregated consumption impulses generated by aggregated yields such as yearly yields in the same way as with the debt impulse function⁷. Its inflection point is assumed to be in τ . The impulses overlap each other, so do the yearly impulses and the entire yearly consumption impulses (GDP). In this way the entire state of the propensity to consume in the economy at a given time will be represented by all parts of the curve, if the curve represents the average propensity to consume generated by all past yields. This will also be the case

⁷ This accords with the rational expectation theory where normal distribution is a standard assumption for future events around theoretical time predictions. In the rational expectation theory it is assumed that economic entities do not make systematic mistakes in predicting future market conditions, that is that the economy in general does not waste information, although random errors are possible and therefore real future events are set around the predictions according to normal distributions run by the C.F. Gauss function. Similarly, we assert that human behavior is somehow genetically determined in a way that consumers' propensities to consume also run along normal distributions around the incomes *ad hoc* onward and therefore around all incomes. The fundamentals of the rational expectations theory can be found in: Muth [7] and Lucas [6].

if the present is in τ . Therefore, for both these functions, integral calculus can be used in formal mathematical analysis. We assume that these two curves are synchronized at τ . The functions represented by these curves run on the demand side of the economy.

Figure 2.2
The consumption impulse function curve



For the supply side, we assume that individual bits of investments from individual bits of savings can be aggregated into lumps of investments associated with individual product life cycles. These in turn can be divided and aggregated into yearly investment aggregates. In the product life-cycle theories, it is generally assumed through experience that products have two epochs in their individual economic history. At first they generate losses. Then they generate profits. This is due to the fact that from their startups for some time the costs are predominant over revenues, which are always the sum of costs and profits, according to the formula:

$$R = C + \Pi \quad (2.1)$$

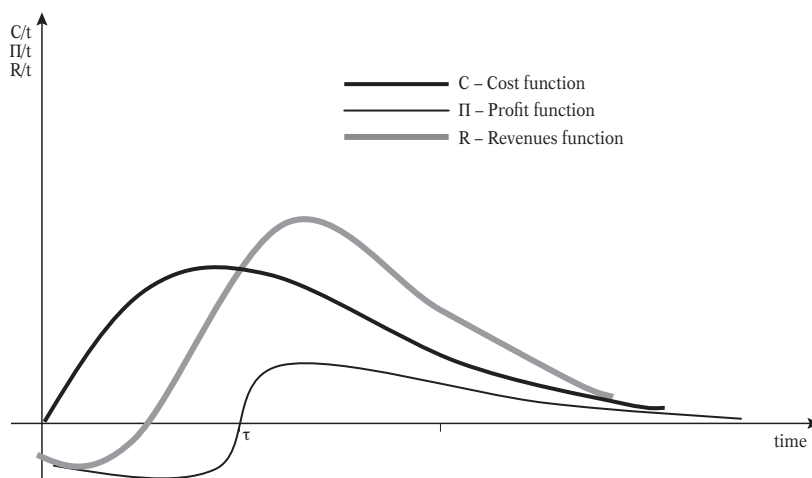
where R stands for revenues, C for costs and Π for profits. This can be also stated for aggregates, particularly yearly aggregates in the entire economy.

For this a system of functions is adopted as in the following graph (Figure 3.2)⁸.

The product life-cycle theories generally state that there are several phases in the development of a product, of which three are adopted here – the innovation phase, the maturing phase and the standardization phase, which overlap each over. The innovation phase finishes at τ . When revenues become positive

⁸ The product life-cycle theory was initially developed by Vernon [14] for international trade. Here, we consider it as a universal feature, not only for international trade.

Figure 3.2
The investment impulse functions curves



(the gray line function passes above the time axis), that is, when sales become predominant over expenditure associated with the product, the maturing phase sets in and lasts until revenues reach their maximum. The product becomes mature and starts to get standardized when it starts generating profits. This happens at τ and if τ is the present and the functions represent yearly aggregates, it is assumed that they also represent the average investment past of the economy in the same way as it was done for the demand side functions presented before. Integrals can also be used.

Given the assumption that the τ period for all of these functions is the same we can integrate the three graphs above into a single one (Figure 4.2)⁹.

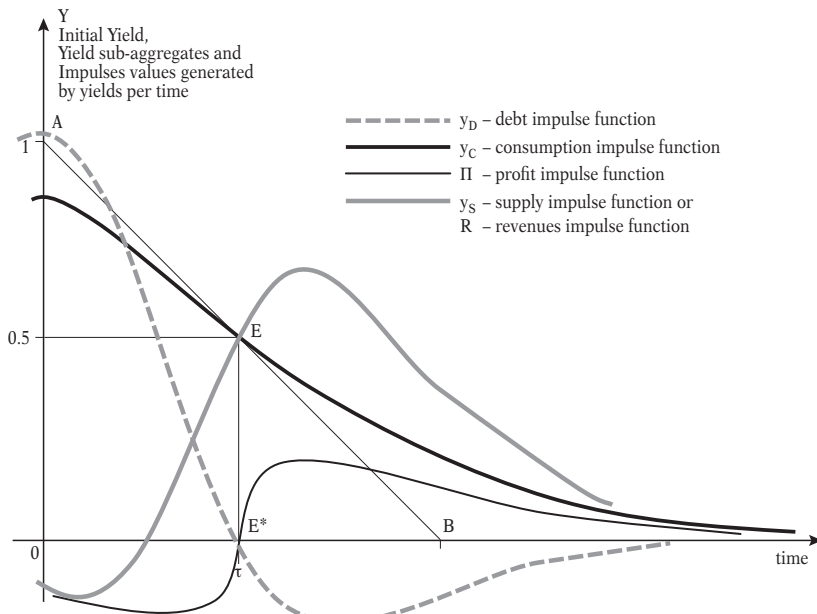
The cost impulse function from graph 3.2 is omitted for simplicity – it runs along the consumption impulse function from τ onward and eventually merges with it as shown further. The following set of equations is to be adopted:

$$\begin{aligned}
 R &= C + \Pi \\
 y_S &= y_C - y_D \\
 R &= y_S
 \end{aligned}
 \tag{2.2}$$

The subtraction of the consumption and the debt impulse functions (y_C and y_D) gives a supply impulse function (y_S), which is identical to the revenues function (R) from graph 3.2. We state that this model is a model of a perfectly open and a perfectly balanced economy – perfectly balanced with abroad and perfectly balanced within. This is because the adoption of the concept of a closed economy is biased. It does not consider the international exchanges (i.e. international trade and international capital movements), which is far

⁹ The graph shows the idea and is not strictly mathematical.

Figure 4.2
The impulse model of the economy



from reality and does not consider the different importance of international exchanges for different economies depending on their sizes. A small and closed economy should have a different economic model mathematically than a big one. The free exchanges act as an equalizer between all economies regardless of their size. In the case of a very big economy, any appropriate closed economy model would tend to be more similar to any appropriate open economy model than in the case of a small economy. This model will be the starting point for further analyses.

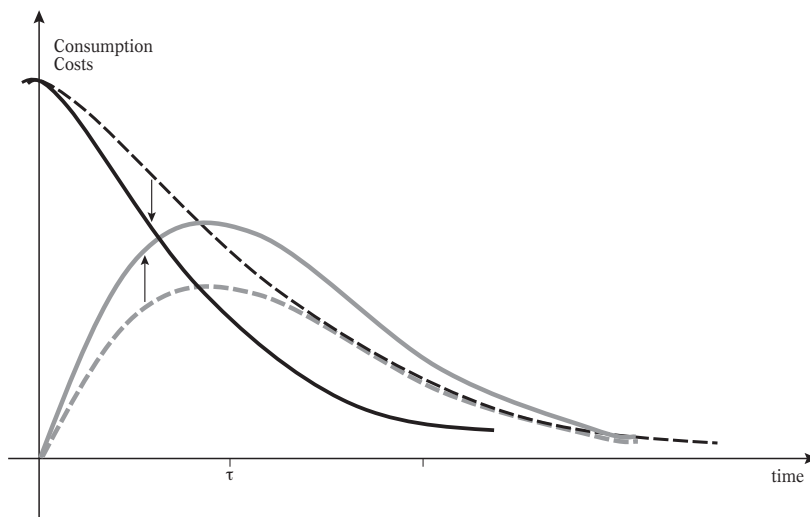
We can assume that the initial yield is spent on consumption or investments and this creates impulses that further drive economic behavior. When the supply impulse function (y_S) trespasses the time axis on the graph, the additional revenues cover the difference between the consumption and the spending wishes of economic entities represented by consumption and debt impulse functions. When the supply impulse function (y_S) trespasses the consumption impulse function (y_C) a profit is generated as the cost impulse function (from graph 3.2 not shown on graph 4.2) runs along the consumption impulse function from τ onward in a perfectly balanced economy. The additional supply over consumption from τ onward is covered by the generated profits. The positive profits from τ onward are in balance with debt as the profit impulse function (Π) from τ onward is a mirror reflection of the debt impulse function (y_D) around the time axis. The latest is in turn a mirror reflection around the vertical axis in τ of the profit impulse function from period $(0, \tau)$. The unba-

lanced negative profits before τ that represent financial investments must be covered by previous yields profit impulses not depleted by debt.

Within this model it is possible to explain downturns. A classical over-investment causes the rise of the costs and the disintegration of the cost impulse function with the consumption impulse function from τ onward, as shown on the graph (the gray line rises):

Figure 5.2

The disintegration of the costs impulse function with the consumption impulse function



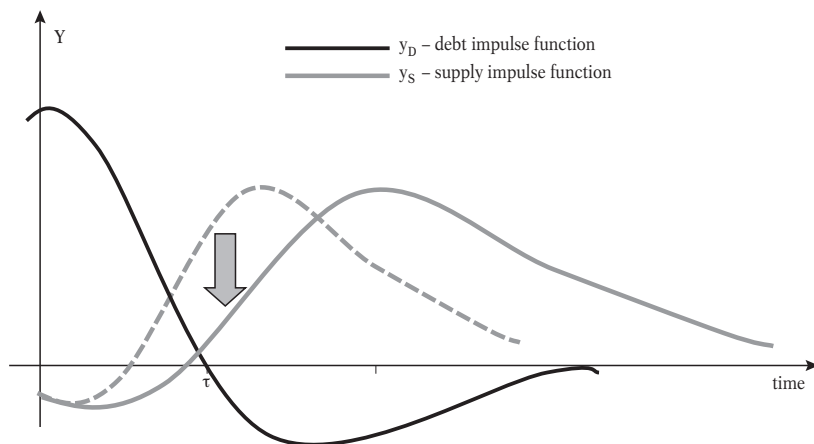
This causes the profit impulse function from Figure 4.2 (which is the subtraction of the supply and cost impulse functions) to fall from τ onward whereas the debt impulse function increases its amplitude (because of investment financed by debt) also from τ onward, and is not anymore counterbalanced by the falling profit impulse function. In the case of consumption disasters¹⁰ (caused by wars, social problems, or other unknown reasons) it is the consumption impulse function which on the graph dives below the costs impulse function. Because the supply impulse function follows downward (because of falling revenues) profits fall and do not counterbalance the debt impulse function.

In the case of a supply shock, as in the 1970s (the high oil prices), the cost impulse function rises and disintegrates with the consumption impulse function from τ onward as well. Then the profit impulse function also falls. In all these situations, because of falling profits, the downward price adjustments are contained, which, in turn, decreases the sales, therefore the supply impulse function falls and the fall of profits continues. This is a resonance situation and the economy may fluctuate in a rhythm modified by casual circumstances.

¹⁰ These are investigated by Barro [4] and Barro & Ursua [2].

In the case of huge long-life capital sector investments (after the oil shocks and before 1992) the supply impulse function disintegrates with other functions from Figure 4.2 and has an extended τ period, as shown in the following graph:

Figure 6.2
The supply impulse function shift



The same happens for the profit impulse function, which does not balance the debt impulse function in the short period.

In the case of huge money expansion (the financial crisis 2008) some of the additional money goes to the demand instead of the supply side of the economy. The debt impulse function increases its amplitude to a greater extent than the profit impulse function, which does not counterbalance it τ onward. If the “absolute” debt is mostly “formal” a financial crisis may accompany and precipitate the economic downturn.

The formal model

The following analysis relates to Figure 4.2. For the consumption impulse function generated by an individual yield we adopt the standard normal distribution run by the C.F. Gauss function. Because microeconomic aggregated probabilities turn to macroeconomic certainties, we use a simpler bell-shaped curve function of the same author instead. But this function does not start from the unity on the yields axis (Y) but from C/Y , and therefore:

$$y_C = \frac{C}{Y} e^{-\frac{t^2}{2\tau^2}} \tag{3.1}$$

As the function starts from C/Y its inflection point is at:

$$y_E = \frac{C/Y}{\sqrt{e}} \tag{3.2}$$

At the same time, since Y is at unity on the axis, we know from the triangle AOB that:

$$y_E = \frac{1}{2} \quad (3.3)$$

The formulae (3.2) and (3.3) give:

$$\frac{C}{Y} = \frac{\sqrt{e}}{2} \approx 0.825 = 82.5\% \quad (3.4)$$

and because $Y = C + I$ (with no Government):

$$\frac{I}{Y} = 1 - \frac{\sqrt{e}}{2} \approx 0.175 = 17.5\% \quad (3.5)$$

From this we can state that a perfectly balanced economy would consume at a level of 82.5% of the entire yield (GDP) and therefore invest at 17.5% of it, which is close to the golden rule¹¹. From (3.1) and (3.4) we have:

$$y_C = \frac{1}{2} e^{\frac{1}{2} - \frac{t^2}{2\tau^2}} \quad (3.6)$$

which is the model's final formula for the perfectly balanced consumption impulse function. For the debt impulse function we will use the Mexican hat function, which, when t is put instead of x , the parentheses are eliminated and the curve trespasses the time axis at τ , takes the shape of:

$$y_D = e^{-\frac{t^2}{2\tau^2}} - \frac{t^2}{\tau^2} e^{-\frac{t^2}{2\tau^2}} \quad (3.7)$$

Subscript D has been added to indicate that it is the debt impulse function. From (2.2), (3.6) and (3.7) we have:

$$y_S = \frac{1}{2} e^{\frac{1}{2} - \frac{t^2}{2\tau^2}} - e^{-\frac{t^2}{2\tau^2}} + \frac{t^2}{\tau^2} e^{-\frac{t^2}{2\tau^2}} \quad (3.8)$$

Subscript S has been added to indicate that it is the supply impulse function. Since, as defined in the previous section, the economy is at all stages of this function and the function represents the average past supply impulse an integral can be used to calculate the total supply impulse at τ ¹²:

$$\varepsilon = \int_0^{\infty} \left(\frac{1}{2} e^{\frac{1}{2} - \frac{t^2}{2\tau^2}} - e^{-\frac{t^2}{2\tau^2}} + \frac{t^2}{\tau^2} e^{-\frac{t^2}{2\tau^2}} \right) dt \quad (3.9)$$

where ε is the ratio between the supply impulse at τ generated by past yields and the average level of past yields at 0 on the time axis, which is at unity on the yields axis. Therefore, it can be used as growth coefficient. This function

¹¹ The golden rule arises from the neoclassical theory of economic growth. See: Solow [11], Barro [3], Barro & Sala-i-Martin [1]. That the balance is set a bit below the optimal level of investments for growth is also expressed in the Ramsey-Cass-Koopmans model, after: Romer [8].

¹² Calculated with the help of: *Derive 6*, Texas Instruments Incorporated.

contains the Mexican hat function of which the integral is equal to 0. This allows:

$$\varepsilon = \frac{1}{2} e^{\frac{1}{2}} \int_0^{\infty} e^{-\frac{t^2}{2\tau^2}} dt \quad (3.10)$$

which is also the integral of the consumption impulse function (3.6) at τ generated by past yields in the same way as for the above-mentioned supply impulse and therefore also the total demand impulse in the situation of its equilibrium with the supply impulse¹³. Using the *gamma* function we have:

$$\varepsilon = \frac{\Gamma\left(\frac{1}{2}\right) \tau \sqrt{2e}}{4} \quad (3.11)$$

and finally:

$$\varepsilon = \frac{\sqrt{2\pi e}}{4} \tau \quad (3.12)$$

This is the growth coefficient over τ period. Therefore, the yearly growth coefficient ε should be:

$$\varepsilon = \frac{\sqrt{2\pi e}}{4} \approx 1.033 = 103.3\% \quad (3.13)$$

This means that a perfectly balanced economy grows by 3.3% a year. The economy may develop quicker at the expense of future development (e.g. financed by debt) or at the expense of other economies. Just as within a given country, where there are regions developing faster at the expense of other regions, this growth shift also happens between the countries, particularly because of the difference in competitiveness. This can be called a time-space shift in the speed of development. Because of (3.4) we can turn (3.13) to:

$$\varepsilon = \frac{\sqrt{2\pi}}{2} \frac{C}{Y} \quad (3.14)$$

and because of (3.5) we can turn it to:

$$\varepsilon = \frac{\sqrt{2\pi e}}{2(2 - \sqrt{e})} \frac{I}{Y} \quad (3.15)$$

These two formulae represent the demand and the supply impulses that compete for the same entire yield Y , so if weighted with α as in the R. Solow's decomposition¹⁴, they can be joined in the formula:

$$\varepsilon = \alpha \frac{\sqrt{2\pi e}}{2(2 - \sqrt{e})} \frac{I}{Y} + (1 - \alpha) \frac{\sqrt{2\pi}}{2} \frac{C}{Y} \quad (3.16)$$

No Solow residual is present in this formula because the model, as based fundamentally on R. Vernon's product life-cycle theory, endogenously contains

¹³ Realized demand is equal to realized supply.

¹⁴ See R. Solow [11].

technological progress. Technological progress is believed here to compete for investments with sheer capital development. Therefore, investments in innovation are in equilibrium with investments in extensive capital development. Instead of the growth of capital stock ΔK as in R. Solow's decomposition we relate in the formula to the level of investments I , because the replacement of old capital by new capital is (as we believe) in equilibrium with extensive new capital development. This means that the "creative destruction" process of J.A. Schumpeter is just as important and in balance with virgin new capital development¹⁵. Because α is taken from the Solow model, it is therefore:

$$\alpha = r \frac{K}{Y} \quad (3.17)$$

where r is the return rate, K the total stock of capital, and Y the annual yield. The possible *market failure* in balancing the different kinds of investments will be expressed in the formula by a lower level of the return rate r .

From (3.16) we can see that the more we invest, the quicker the economy grows on condition that the return rate does not drop substantially. This also means that investments driven by a low Central Bank interest rate may become low-profitability investments and cause the profit impulse function from Figure 4.2 to drop. The formula (3.16) can therefore warn against downturn impulses.

Because all the supply impulses from before the τ period turn into demand impulses in the last τ period (because of falling prices) and all the demand impulses from before the τ period turn into supply impulses in the last τ period (because of rising prices) the last τ period contains all the past inertia of the economy. This allows us to turn (3.16) into:

$$\varepsilon = \alpha \frac{\sqrt{2\pi e}}{2(2 - \sqrt{e})} \frac{I_\tau}{Y_\tau} + (1 - \alpha) \frac{\sqrt{2\pi}}{2} \frac{C_\tau}{Y_\tau} \quad (3.18)$$

and in order to discount the capital volatility over the short period we will turn (3.17) into:

$$\alpha = r \frac{\bar{K}_\tau}{\bar{Y}_\tau} \quad (3.19)$$

where subscript τ indicates that these values are over the τ period and in the formula (3.19) they are average yearly values over the last τ period. This means that what happened in the economy over the last τ period – the last few years in terms of economic past – determines the level of next year's economic growth. This real *endogenous growth engine* coefficient reflects the idea of economic inertia as in the case of the flywheel of an engine. The coefficient should be inserted into the formula:

$$Y^* = \varepsilon Y \quad (3.20)$$

¹⁵ See J.A. Schumpeter [10].

where Y is the last year's and Y^* the next year's entire yield. This is a model of a perfectly open economy that is perfectly balanced with abroad as far as the balance of payments is considered and is free from any central interventions. In the real economy, however, there can be imbalances in international capital flows and in international trade and also imbalances within the economy, which are dealt with in the next section.

Interventions and BP imbalances

Central interventions will to a large extent be expressed invisibly in the above model. That is, when they are effective or counter-effective in the longer term they translate into changed values in the (3.16) and (3.18) formulae. Therefore, we consider that the impact of the state that needs to be additionally accounted for is only present for a year before it gets diluted in the market in the second year (when expectations, whether rational or adaptive, reach their full force). This is in part due to the fact that budgets are usually set annually and only for the next year and that the budget for the subsequent year is usually set in the preceding year. The same is assumed here for the Central Bank and the balance of payments impacts. More generally, we analyze the impact on the economy of huge bodies and usually there are three of them – the Government, the Central Bank, and the “Abroad”¹⁶.

We now engage in IS-LM-BP model modifications to tailor them to the main model above, and we assume in this calculus that whatever impact is not present in the above model of the *endogenous growth engine* it will impact the economy only within the next year. We adopt the following IS curve form:

$$r = \frac{A}{\alpha} - \frac{Y}{\alpha m} \quad (4.1)$$

and the following LM curve form:

$$r = \frac{k}{\beta} Y - \frac{M}{\beta P} \quad (4.2)$$

where r is the interest rate. The values on the right-hand side of the above two formulae are so well reported in the Keynesian literature (although different symbols may be used) that we will limit our comments to what is germane to this discussion. Those functions are often more developed in detail (e.g. A – autonomous demand is often divided in its components) but this is not necessary here. Merging the two formulae we have:

$$\frac{A}{\alpha} - \frac{Y}{\alpha m} = \frac{k}{\beta} Y - \frac{M}{\beta P} \quad (4.3)$$

k is the reciprocal of V , the velocity of money circulation:

¹⁶ We consider the state as some kind of alien body run by different laws, but we are not as harsh on the state as Rothbard is. See [9].

$$k = \frac{1}{V} \quad (4.4)$$

which is present in the *I. Fisher* formula:

$$MV = PT \quad (4.5)$$

As we are interested in economic growth and not the overall level of transactions, we will use the formula in the form for final transactions, without the intermediate ones:

$$MV = PQ \quad (4.6)$$

where Q is the physical volume of production (where as T was the number of transactions). Therefore, we can turn the (4.3) formula into:

$$\frac{A}{\alpha} - \frac{Y}{\alpha m} = \frac{MY}{\beta PQ} - \frac{M}{\beta P} \quad (4.7)$$

Since $Y = PQ$ we have:

$$\frac{A}{\alpha} - \frac{Y}{\alpha m} = \frac{M}{\beta} - \frac{M}{\beta P} \quad (4.8)$$

Solving the above equation with respect to Y gives:

$$Y = m \left(A - \frac{\alpha}{\beta} M + \frac{\alpha}{\beta} \frac{M}{P} \right) \quad (4.9)$$

We assume that the sensitivities α and β (that are often divided into components) are equal because investments and savings are basically equal too and this is because of the liquidity of today's financial markets. If this is not the case, it is because of the foreign markets and we assume that this effect is fully discounted by the impact of the BP analyzed further. Short-term currency fluctuations tend to correct each other, while long-term currency shifts prompt economic entities to turn to other kinds of money. If this is so, we have:

$$Y = m \left(A - M + \frac{M}{P} \right) \quad (4.10)$$

As we do not explain the whole economy working within the Keynesian model but only the impacts over the *endogenous growth engine*, we state that only the changes in the values from (4.10) are important here¹⁷:

$$\Delta Y = m(\Delta A - \Delta M_N + \Delta M_R) \quad (4.11)$$

¹⁷ Because the changes are not depleted by expectations in the short term.

and we have added subscripts to discriminate nominal (N) and real (R) values. The above change should be endeared because it is easier to define the growth of the so-called autonomous demand ΔA than the total autonomous demand. The same is with the money levels M_N and M_R . It is certainly easier to define the actual growth of money generated by Central Bank issues than the entire level of money in an economy because of its many definitions (that is, the different aggregates of money according to their liquidities) and because it is combined with the velocity of money circulation in formulae (4.5) and (4.6) in a way that it can be discounted by it. The change of autonomous demand follows the intervention of the government and/or the central bank:

$$\Delta A = \Delta G + \Delta M_N \quad (4.12)$$

where ΔG stands for government change of expenditure (the Government is defined here as all tax-financed institutions, including local governments) and ΔM_N for Central Bank nominal change of money issuance. If we put together (4.11) and (4.12) we have:

$$\Delta Y = m(\Delta G + \Delta M_R) \quad (4.13)$$

This means that both the Government and the Central Bank act through the multiplier m , when changing their policy. When increasing expenditure, the Government increases the velocity of money circulation V in the (4.5) and the (4.6) formulae, whereas the Central Bank increases the level of money M , of which only real money M_R is of importance. Government interventions can effectively increase the next year's growth over the *endogenous growth engine* only when it is an increased expenditure (not the continuation of the previous year's high expenditure). Additional money can also flow into the economy from abroad; therefore, the increase in the balance of payments needs to be inserted into the equation (4.13):

$$\Delta Y_i^* = m(\Delta G + \Delta M_R + \Delta BP) \quad (4.14)$$

Subscript i shows the additional growth level generated by current intervention and imbalances. The asterisk indicates the next year's value. We state here that all three above-mentioned huge bodies may generate additional growth through the multiplier m . These impacts are, however, counteracted by the fact that tax increases and debt deplete the multiplier and tax increases, debt and inflation deplete the *endogenous growth engine*. Depending on the situation and on the span of time, the interventions can be positive, indifferent or negative and the values in (4.14) can be either positive or negative, except the multiplier. For the multiplier to be effective in fostering growth, it needs to be far above 1. Formula (4.14) should be inserted into formula (3.20) from the previous section as follows:

$$Y^* = \varepsilon Y + \Delta Y_i^* \quad (4.15)$$

Now we need to address the multiplier. The original version of the Keynesian multiplier for an open economy with the state has already been modified to:

$$m = \frac{1}{1 - \frac{\Delta C}{\Delta Y}(1 - t) + \frac{\Delta Im}{\Delta Y}} \quad (4.16)$$

where Im stands for imports. The tax level t is the proportion of Y and is therefore no higher than 1. The Government, when only redistributing citizens' incomes, competes for labor with the private sector. Therefore it decreases the private sector's profits. It also does so when directly taxing the profits through CIT . Indirect taxes should be included as well.

The public sector also redistributes between the supply and demand sides of the economy. The "taxes aggregate" position on the supply side can be expressed by the following formula:

$$T_S = CIT + \alpha IT + d_I \Delta D_G - d_I R \quad (4.17)$$

where CIT is the corporate income taxes total aggregate, α is the capital remuneration share in the GDP , IT is the indirect taxes total aggregate (with e.g. VAT). This indirect taxation is believed to impact the supply side in proportion to its share in the economy. d_I is the share of capital investment debt in the entire growth of debt of the private sector, ΔD_G is the yearly growth of Government debt, which competes with the private sector. R is the aggregated interest paid by the Government on its debt. The taxes aggregate position on the demand side can be expressed by:

$$T_D = PIT + (1 - \alpha) IT + d_C \Delta D_G - d_C R \quad (4.18)$$

where PIT is the personal income taxes total aggregate, $(1 - \alpha)$ is the share of wages in the GDP , d_C is the share of consumer debt in the overall growth of debt of the private sector. In order for the Government not to redistribute between the demand and the supply sides, the proportion between these two values should be the same as the proportion between Government expenditures on the demand and the supply sides:

$$\frac{G_D}{G_S} = \frac{PIT + (1 - \alpha) IT + d_C \Delta D_G - d_C R}{CIT + \alpha IT + d_I \Delta D_G - d_I R} \quad (4.19)$$

where G_D and G_S are government expenditures on the demand and the supply sides respectively. G_D are, in general, public staff remuneration and social support. G_S are, in general, public procurement and direct government business support. This can be turned into a redistribution coefficient:

$$\rho = \frac{G_S(PIT + (1 - \alpha) IT + d_C \Delta D_G - d_C R)}{G_D(CIT + \alpha IT + d_I \Delta D_G - d_I R)} \quad (4.20)$$

Finally, the above coefficient can be inserted into the multiplier:

$$m = \frac{1}{1 - c(1 - t)\rho + i} \quad (4.21)$$

where $c = \Delta C/\Delta Y$ and $i = \Delta Im/\Delta Y$ are the propensities to consume and for imports respectively. It is convenient to combine (4.14) with (4.21):

$$\Delta Y_i^* = \frac{\Delta G + \Delta M_R + \Delta BP}{1 - c(1 - t)\rho + i} \quad (4.22)$$

When analyzing the near past it is easy to put in formula (4.14) the reported ΔBP value. However, when forecasting we need to make a calculus that will be based on the IS-LM-BP model. It is needed because the change in the near-future balance of payments ΔBP^* has a direct impact on the present growth level: it is exogenous money that is spent directly (a similar thing may happen with Government expenditure financed by money issuance $\Delta G_{\Delta M}$). Basically, the BP level can be expressed by the following formula:

$$BP = X + BK \quad (4.23)$$

where X is the net exports balance and BK stands for the balance of capital movements. This is developed in the IS-LM-BP model as:

$$BP = Ex_A - Im_A - iY + \gamma(r - r_F) \quad (4.24)$$

The right-hand side of this formula is compared to 0 and when solved with respect to r gives the BP curve of the IS-LM-BP model. This is well reported in the literature, although other symbols may be used and the values in the (4.24) formula are often divided into components, but this is not necessary here. Exports, however, which are considered to be autonomous here – Ex_A – can be divided into autonomous and driven by foreign yearly yields changes in the same way as in the case of imports for the given economy. The foreign partner economies can be summarized in this respect as follows:

$$Ex = \sum_{F=1}^n (Ex_{AF} + e_F Y_F) \quad (4.25)$$

where Ex_{AF} , e_F and Y_F stand for autonomous exports to specific countries, propensities for exports to specific countries, and the yield (GDP) levels of the mentioned countries respectively. If we divide the imports in the same way, we will have the following formula for the balance of trade:

$$X = \sum_{F=1}^n Ex_{AF} + \sum_{F=1}^n e_F Y_F - \sum_{F=1}^n Im_{AF} - \sum_{F=1}^n i_F Y \quad (4.26)$$

If we relate the propensity for imports not to our economy but to a given foreign economy, we can replace Y by Y_F . This allows for:

$$X = \sum_{F=1}^n X_{AF} + \sum_{F=1}^n x_F Y_F \quad (4.27)$$

where X_{AF} is the autonomous net export balance to a given foreign economy, x_F is the propensity for net export balance to a given foreign economy. This change is justified as the autonomous net export balance is less volatile than exports and imports taken separately. The same is true of the propensities. The autonomous net exports are fixed values and they can be aggregated into X_A . We assume that the long-term changes in the balance propensities (Δx_F) are technology and long-term investments driven and are inherited in the next short period of time from the past:

$$X^* = X_A + \sum_{F=1}^n (x_F + \Delta x_F) Y_F \quad (4.28)$$

where X^* is the predicted value of next year's net exports. The steep changes in growth volatilize the trade balance so we put in the formula their ratios:

$$X^* = X_A + \sum_{F=1}^n (x_F + \Delta x_F) Y_F^* \frac{\Delta Y_F^*}{\Delta Y_F} \frac{\Delta Y}{\Delta Y^*} \quad (4.29)$$

and finally to extract solvable values:

$$X^* = X_A + \sum_{F=1}^n (x_F + \Delta x_F) Y_F^* \frac{\Delta Y (Y_F^* - Y_F)}{\Delta Y_F (Y^* - Y)} \quad (4.30)$$

The asterisks indicate next year's future values. Y^* can be solved within the whole set of equations presented in this work. The same is with Y_F^* if we have the same sets of these equations for each of the foreign partner economies. The unknown value of autonomous exports¹⁸ can be calculated from past experience as it is stable:

$$X_A = X - \sum_{F=1}^n x_F Y_F \frac{\Delta Y_{(-1)}}{\Delta Y_{F(-1)}} \frac{\Delta Y_F}{\Delta Y} \quad (4.31)$$

Now we will analyze the capital imbalance BK expressed initially in (4.24) by:

$$BK = \gamma(r - r_F) \quad (4.32)$$

which is often divided into components, but it is not necessary here. We believe that this formula is biased as far as economic growth is considered, because the interest rates (r and r_F) deplete Tobin's q . As we are not interested in speculative capital movements here but in the relative starving for investment capital between different economies, we will convert the formula into:

¹⁸ We believe that treating the autonomous values as the previous year's values is wrong. In this work, autonomous means the fixed part based on contracts and supply chains. On the other hand, the non-autonomous values are the market-volatile parts of larger aggregates.

$$BK^* = \sum_{F=1}^n \beta_F (q^* - q_F^*) \quad (4.33)$$

The asterisk denotes a future value. β_F is a barrier coefficient with a given country, and the other right-hand side values of the above equation are Tobin's q and Tobin's q_F , one for the given economy, the other for the given partner economies. Although Tobin's q is volatile and in line with the business cycle, the differences between the q_s of different countries are quite stable, so we can put in the forecast the present values for Tobin's q in case we are unable to predict them (above the market it is the given country's Central Bank which is the q level maker). In smaller countries, foreign investment also depends on negotiations between the foreign investors and the given country's government, which is also run by the human factor. Otherwise, the barrier coefficient is stable and can be calculated for the given foreign country on the basis of past experience in the following way:

$$\beta_F = \frac{BK_F}{q - q_F} \quad (4.34)$$

where BK_F is the balance of payments with a given partner country. We will now combine formulae (4.30), (4.33) and (4.34) into a single equation for the forecast BP^* :

$$BP^* = X_A + \sum_{F=1}^n \left[(x_F + \Delta x_F) Y_F^* \frac{\Delta Y (Y_F^* - Y_F)}{\Delta Y_F (Y^* - Y)} + BK_F \frac{(q^* - q_F^*)}{(q - q_F)} \right] + \delta_{BP} \quad (4.35)$$

where δ_{BP} is the difference between the calculated and the real BP . As for the near past, i.e. the previous year, the (4.35) formula should be:

$$BP = X_A + \sum_{F=1}^n \left[x_F Y_F \frac{\Delta Y_{(-1)}}{\Delta Y_{F(-1)}} \frac{\Delta Y_F}{\Delta Y} + BK_F \right] + \delta_{BP} \quad (4.36)$$

which allows us to calculate the systematic mistake:

$$\delta_{BP} = BP - X_A - \sum_{F=1}^n \left[x_F Y_F \frac{\Delta Y_{(-1)}}{\Delta Y_{F(-1)}} \frac{\Delta Y_F}{\Delta Y} + BK_F \right] \quad (4.37)$$

Now we need to subtract BP reported in the past year from that forecast for the next year:

$$\Delta BP^* = BP^* - BP \quad (4.38)$$

in order to calculate the direct present impact of the BP on the growth. This in turn should be inserted in equation (4.15):

$$Y^* = \varepsilon Y + \Delta Y_i^* + \Delta BP^* + \Delta G_{\Delta M}^* + \delta_{\Delta Y} \quad (4.39)$$

where we have also inserted Government expenditure financed by money issuance $\Delta G_{\Delta M}^*$ that may happen in some cases when the Central Bank is not fully

independent. This kind of intervention, though very effective in the short run, increases the “absolute” debt by amplifying the debt impulse curve on graph 4.2, which is expressed in the formula for the *endogenous growth engine* (3.18) in lower component values. In the case of ΔBP^* the exogenous money usually goes entirely to the supply side of the economy, therefore not causing such harm in the longer term. Normally, we can omit $\Delta G_{\Delta M}^*$. $\delta_{\Delta Y}$ is the systematic mistake that can be calibrated on the basis of past experience:

$$\delta_{\Delta Y} = Y - \varepsilon_{(-1)}Y_{(-1)} - \Delta Y_i - \Delta BP + \Delta G_{\Delta M} \quad (4.40)$$

Below is the summary of the above analyses.

Summary and long-term growth

The following is a set of 10 equations for the forecast next year’s economic growth. Values with the asterisks are future yearly values. Values subscripted with τ are values over the last τ period. Other values are present values, i.e. over the last year. The τ period may be calibrated for the given economy on the basis of past experience. If not, then we can assume that the τ period is equal to 4 years. Future Y^* and Y_F^* can be solved within this set of equation and the given partner economies’ sets of equations. If we cannot predict the future q^* and q_F^* values that depend on the Central Bank’s current policy, we use last year’s values instead. We may usually assume that the Government expenditure will not be suddenly financed by money issuance $\Delta G_{\Delta M}^*$. We keep in mind that in smaller countries foreign investments may depend strongly on negotiations between the foreign investors and the given country’s government and this may be added into the equations concerning the balance of payments.

$$Y^* = \varepsilon Y + \Delta Y_i^* + \Delta BP^* + \Delta G_{\Delta M}^* + \delta_{\Delta Y} \quad (4.39)$$

$$\varepsilon = \alpha \frac{\sqrt{2\pi e}}{2(2 - \sqrt{e})} \frac{I_\tau}{Y_\tau} + (1 - \alpha) \frac{\sqrt{2\pi}}{2} \frac{C_\tau}{Y_\tau} \quad (3.18)$$

$$\Delta Y_i^* = \frac{\Delta G + \Delta M_R + \Delta BP}{1 - c(1 - t)\rho + i} \quad (4.22)$$

$$\Delta BP^* = BP^* - BP \quad (4.38)$$

$$\delta_{\Delta Y} = Y - \varepsilon_{(-1)}Y_{(-1)} - \Delta Y_i - \Delta BP + \Delta G_{\Delta M} \quad (4.40)$$

$$\alpha = r \frac{\bar{K}_\tau}{\bar{Y}_\tau} \quad (3.19)$$

$$\rho = \frac{G_S(PIT + (1 - \alpha)IT + d_c \Delta D_G - d_c R)}{G_D(CIT + \alpha IT + d_l \Delta D_G - d_l R)} \quad (4.20)$$

$$BP^* = X_A + \sum_{F=1}^n \left[(x_F + \Delta x_F) Y_F^* \frac{\Delta Y(Y_F^* - Y_F)}{\Delta Y_F(Y^* - Y)} + BK_F \frac{(q^* - q_F^*)}{(q - q_F)} \right] + \delta_{BP} \quad (4.35)$$

$$X_A = X - \sum_{F=1}^n x_F Y_F \frac{\Delta Y_{(-1)}}{\Delta Y_{F(-1)}} \frac{\Delta Y_F}{\Delta Y} \quad (4.31)$$

$$\delta_{BP} = BP - X_A - \sum_{F=1}^n \left[x_F Y_F \frac{\Delta Y_{(-1)}}{\Delta Y_{F(-1)}} \frac{\Delta Y_F}{\Delta Y} + BK_F \right] \quad (4.37)$$

The growth of an economy over the next year is determined by a combination of three impacts – one from a more distant past, one from the last year, and one from the present, which is expressed in the first formula of the set. The second formula for the *endogenous growth engine* is the main growth inertia of the economy. The third formula represents the effects of the last year's interventions and imbalances not covered by the previous formula. The fourth formula is a forecast of the direct impact of change in the balance of payments. The fifth formula is the main systematic mistake. The remaining formulae are for component values that have to be calculated, since they are not reported. One of them is the systematic mistake on the predicted balance of payment.

The systematic mistakes' formulae are a necessary element of the theory. They express three kinds of bottlenecks present in every economy. The first is the infrastructure bottleneck. The development of a given economy can be contained by capacity limitations of the transportation system that can arise from natural limitations and the lack of investment in the infrastructure because of the lack of financing. Regardless of the impulses, the economy will not develop in a box. The second bottleneck is the institutional bottleneck. No matter the impulses, the badly organized institutions with wrong administrative priorities are able to restrain economic activity. The third bottleneck is the innovation attitudes' bottleneck (cultural bottleneck). No matter the impulses, if in the given economy there are no innovation carriers and entrepreneurial entities, they will not coalesce into economic growth. These three bottlenecks are inherited from the past and determine long-term economic performance. The infrastructure, the institutions and the innovative business culture determine the different levels of economic performance of different countries in the longer term. The systematic mistakes' formulae also contain the repeated errors in data collection and possibly other durable influences of unknown origin. Nonetheless, the level of the main systematic mistake (4.40) is a mean to compare the economic condition of different countries as far as their potential is considered.

To speed up long-term growth we should invest more, but only in a profitable way, develop the infrastructure, optimize the institutions, and spread innovative attitudes. We can also slow down the depreciation of the future stock of capital (not yet being subject to the present "creative destruction" process) by managing it in a better way and by choosing longer life-cycle better-quality technologies in some sectors (for example the electric power

sector). We can also slow the depreciation of consumer goods by prolonging their life cycle through better quality. This will ensure greater future savings for investments and enhance the present level of investment as better quality will create new space for investments in economies saturated with capital and prevent premature capital demise in new quickly developing economies. This process, aimed at future capital, competes for new investments with the "creative destruction" process aimed at the past capital saturation problem. The "creative destruction" process can be enhanced by the towns' gentrification process. This returns to the economy the best locations without forcing it to withdraw from the environment more new space in usually worse locations. Exploring sectors for capital durability and quicker capital replacement may be the only way to speed up developed economies that do not possess any revolutionary technologies to introduce. An audit could be done of a given economy to browse for growth resources.

This theory is devised as a tool to weigh market forces with central interventions and to predict downturns, and to optimize economic policy. The possible empirical verification of the model should not be very difficult. Because the model will be verified on past experience, we can use at first the reported value of ΔBP^* , thus avoiding the need to solve complicated equations. After that, also using the reported values for the symbols with asterisks, we can verify the remaining equations associated with international economics.

When analyzing the economy only in the longer term, for a general conclusion we can use formulae (3.16) and (3.17) instead of (3.18) and (3.19), and combining them we have:

$$\varepsilon = \frac{\sqrt{2\pi}}{2} \left[r \frac{K}{Y} \left(\frac{\sqrt{e}}{2 - \sqrt{e}} \frac{I}{Y} - \frac{C}{Y} \right) + \frac{C}{Y} \right] \quad (5.1)$$

Multiplying by Y and given that $Y^* = \varepsilon Y$ in the longer term, we have:

$$Y^* = \frac{\sqrt{2\pi}}{2} \left[r \kappa \left(\frac{\sqrt{e}}{2 - \sqrt{e}} I - C \right) + C \right] \quad (5.2)$$

where κ is a rather constant capital to output ratio that can be reported empirically. We can see that in the longer term the growth of an economy is a function of the return rate r (the real interest rate), investments I , and consumption C .

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ENDOGENICZNE I EGZOGENICZNE CZYNNIKI WZROSTU GOSPODARCZEGO

Streszczenie

Celem artykułu jest wyjaśnienie udziału głównych czynników determinujących poziom wzrostu gospodarczego zarówno w krótkim i jak i w długim okresie. Analiza dotyczy gospodarki otwartej, inicjalnie zrównoważonej wewnętrznie i zewnętrznie. Przyjęto założenie, że inwestycje czysto kapitałowe są zrównoważone z inwestycjami w innowacje oraz że inwestycje w nowy kapitał są zrównoważone z inwestycjami w wymianę starego kapitału. W odróżnieniu od czynników endogenicznych, które oddziałują na wzrost gospodarczy w sposób zbliżony, ale nie identyczny, do neoklasycznego, czynniki egzogeniczne w znacznym stopniu mają charakter mnożnikowej interwencji, którą można opisać przy pomocy silnie zmodyfikowanego modelu IS-LM-BP. Oprócz inercji gospodarczej działającej w dłuższym okresie oraz krótkookresowego działania mnożnikowego, występuje jeszcze natychmiastowa stymulacja, którą należy uwzględnić w modelu.

W artykule przedstawiono podstawowy model bazujący od strony popytu na funkcji długu według autorskiego pomysłu oraz na funkcji konsumpcji inspirowanej w pewnym zakresie przez teorię racjonalnych oczekiwań. Od strony podaży model bazuje na funkcjach kosztu, przychodów i zysku inspirowanych przez teorię cyklu życia produktu według R. Vernona. Funkcje te synchronizuje się ze sobą i łączy w jeden wspólny model, do którego inkorporuje się zmodyfikowany formalnie model IS-LM-BP.

Podstawowy wniosek z przeprowadzonej analizy można sprowadzić do tego, że gospodarka taka, jaka obecnie funkcjonuje, jest tworem heterogenicznym. Jednocześnie jest rynkowa w tym sensie, iż modelowo można utożsamić ze sobą wszystkie rodzaje inwestycji, ale jednocześnie jest poddana wpływowi, które według ujęcia neoklasycznego nie mają charakteru rynkowego.

Słowa kluczowe: endogeniczny, egzogeniczny, wzrost, gospodarka, interwencja

Kody JEL: E12, E13, E17, E63, F41, F43, F47, H12, O11, O41
