Appendix

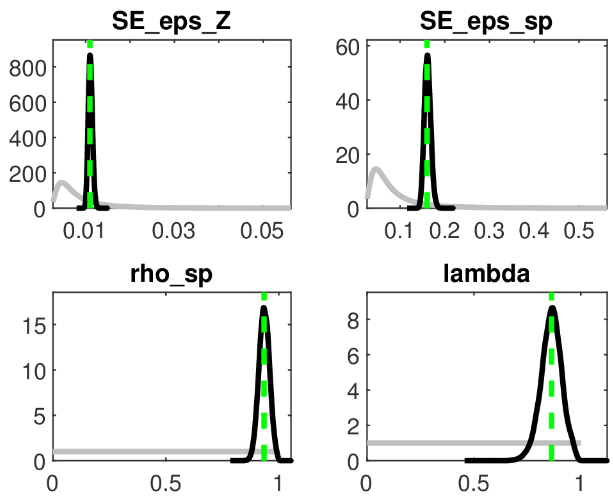
Additional tables and figures

Prior distributions of parameters

Parameter	Description	Distribution shape	Mean	Std. dev.
λ	Calvo wage contract prob.	Uniform [0, 1]	0.5	0.289
ρ_z	Autocorr. of prod. process	Uniform [0, 1]	0.5	0.289
σ_z	Std. dev. of prod. shock	Inverse Gamma	0.01	∞
ρ_{sp}	Autocorr. of spread process	Uniform [0, 1]	0.5	0.289
σ_{sp}	Std. dev. of spread shock	Inverse Gamma	0.1	∞

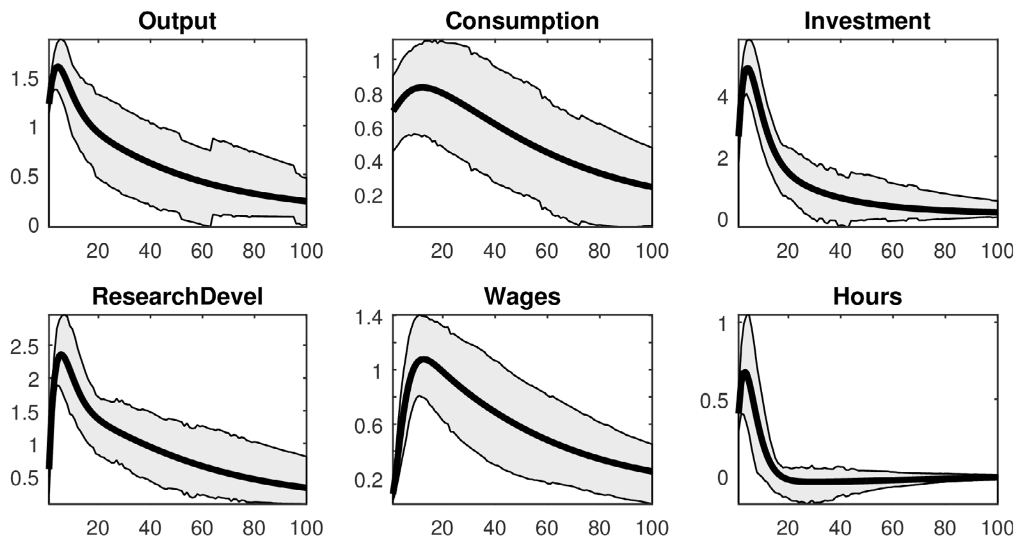
Source: Own calculations.

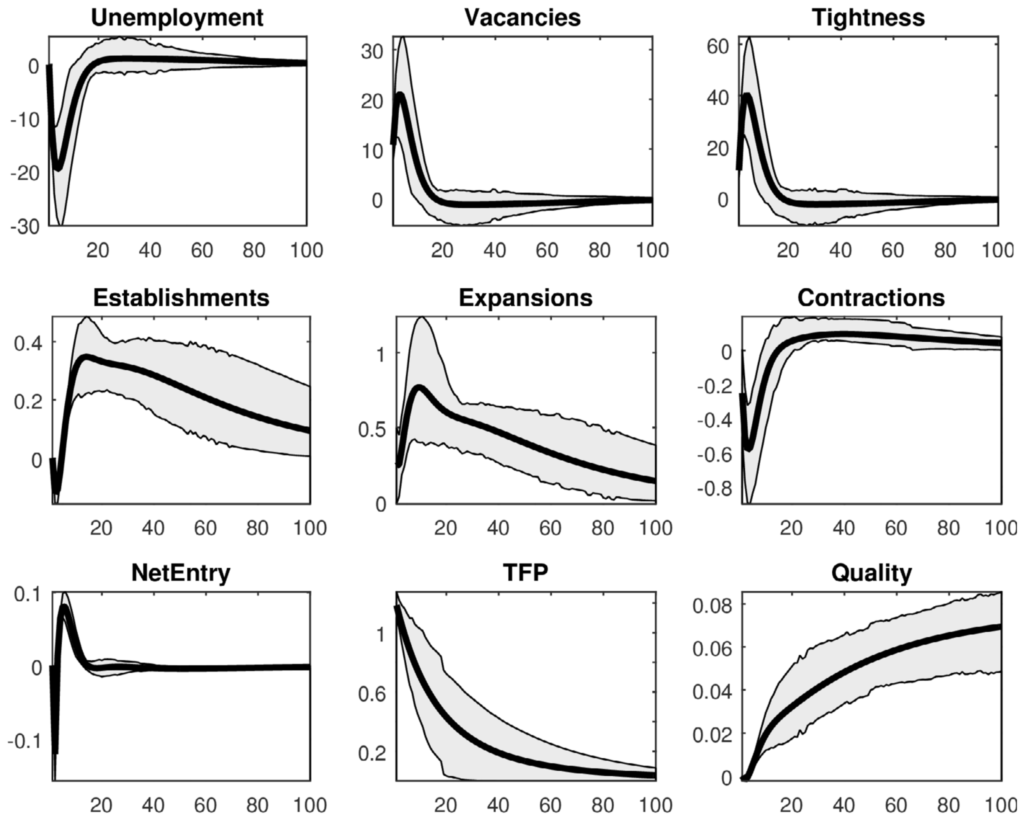
Prior and posterior distributions of estimated parameters



Source: Own calculations.

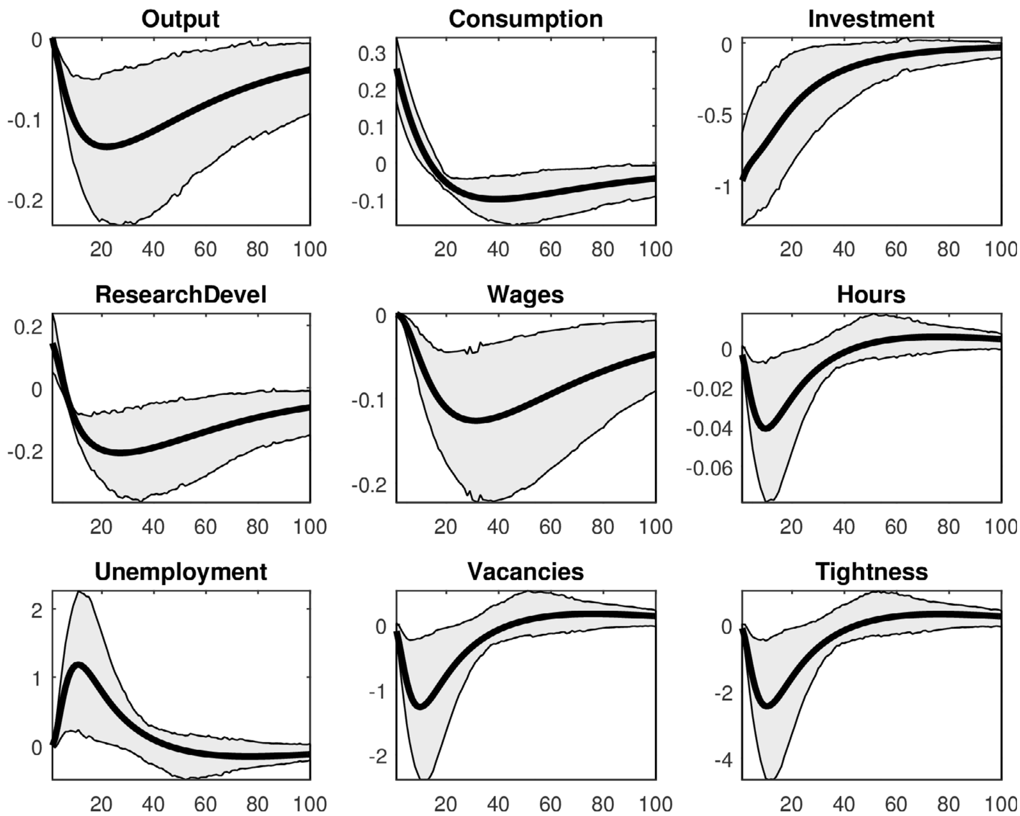
Bayesian impulse response functions to productivity shock

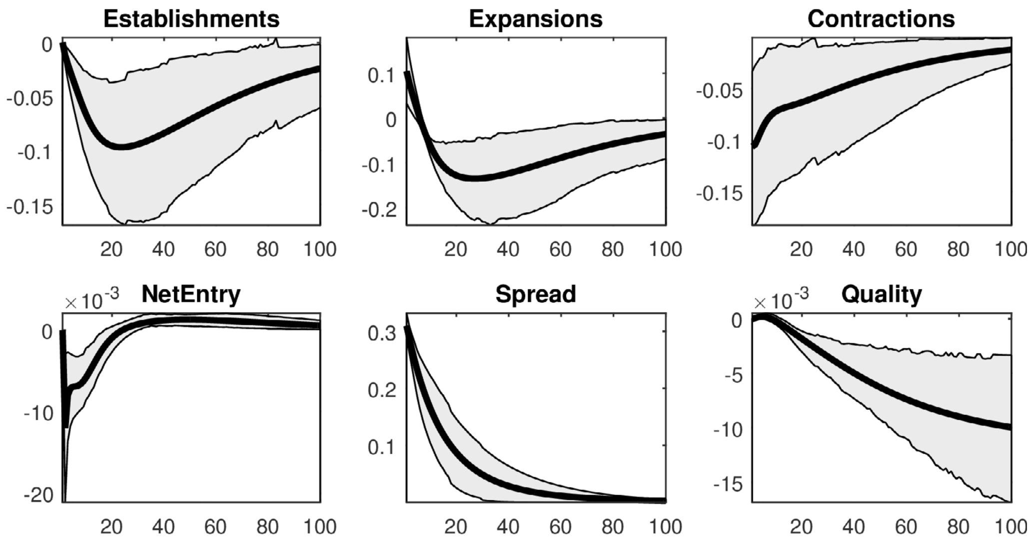




Source: Own calculations.

Bayesian impulse response functions to interest rate spread shock





Source: Own calculations.

Full set of stationarised model equations

Stationarised variables notation

$$\hat{X}_t \equiv X_t / Q_t$$

Stationarising variables

$$g_t^Q \equiv Q_{t+1} / Q_t = \eta_t^{\frac{1}{(1-\alpha)(\sigma-1)}} \tag{1}$$

$$\gamma_{t,t+1} \equiv Y_{t+1} / Y_t = g_t^Q \cdot \hat{Y}_{t+1} / \hat{Y}_t \tag{2}$$

Incumbents' problem

$$\phi_t = 1 \tag{3}$$

$$v_t = A_t + B_t \phi_t \tag{4}$$

$$\pi_t = \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1-\chi_t} \right) \phi_t - \left(1+\zeta r_t^l\right) \omega_t f \tag{5}$$

$$A_t + B_t \phi_t = \pi_t + \mathrm{E}_t \left[\Lambda_{t,t+1} \left(1-\delta_t\right) \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\chi_t(\iota-1)+1}{\eta_t} \phi_t \right) \right] \tag{6}$$

$$0 = -\left(1+\zeta r_t^l\right) \frac{\omega_t}{a} \frac{1}{\left(1-\chi_t\right)^2} + \mathrm{E}_t \left[\Lambda_{t,t+1} \left(1-\delta_t\right) \gamma_{t,t+1} B_{t+1} \frac{(\iota-1)\phi_t}{\eta_t} \right] \tag{7}$$

$$B_t = \left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t}{1-\chi_t} \right) + \mathrm{E}_t \left[\Lambda_{t,t+1} \left(1-\delta_t\right) \gamma_{t,t+1} B_{t+1} \frac{\chi_t(\iota-1)+1}{\eta_t} \right] \tag{8}$$

Entrants' problem

$$v_t^e = -\left(1+\zeta^e r_t^l\right) \omega_t \left(f^e + \frac{1}{a^e} \frac{\chi_t^e}{1-\chi_t^e} \right) + \chi_t^e \mathrm{E}_t \left[\Lambda_{t,t+1} \gamma_{t,t+1} \left(A_{t+1} + B_{t+1} \frac{\sigma}{\sigma-1} \phi_{t+1} \right) \right] \tag{9}$$

$$0=-\left(1+\zeta^{e} r_t^{l}\right)\frac{\omega_t}{a^{e}}\frac{1}{\left(1-\chi_t^{e}\right)^{2}}+\mathrm{E}_t\left[\Lambda_{t,t+1}\gamma_{t,t+1}\left(A_{t+1}+B_{t+1}\frac{\sigma}{\sigma-1}\phi_{t+1}\right)\right] \tag{10}$$

$$v_t^{e}=0 \tag{11}$$

Establishment dynamics

$$\delta_t=1-\left(1-\delta^{exo}\right)\left(1-M_t^{e}\right) \tag{12}$$

$$\left(1+\zeta r_t^{l}\right)\frac{\omega_t}{a}\frac{\chi_t}{1-\chi_t}\phi_t^{*}=\mathrm{E}_t\left[\Lambda_{t,t+1}\left(1-\delta_t\right)\gamma_{t,t+1}\left(A_{t+1}+B_{t+1}\frac{\chi_t(\iota-1)+1}{\eta_t}\phi_t^{*}\right)\right] \tag{13}$$

$$M_t^{x}=M_t\left(1-\chi_{t-1}\right)\left(1-\frac{\phi_{t-1}^{*}}{\phi_t^{*}\eta_{t-1}}\right) \tag{14}$$

$$M_{t+1}=\left(1-\delta_t\right)\left(M_t-M_t^{x}\right)+M_t^{e} \tag{15}$$

$$\eta_t=\left(1-\chi_t+\chi_t\iota\right)\left(1-\frac{M_t^{e}}{M_{t+1}}+\frac{M_t^{e}}{M_{t+1}}\frac{\sigma}{\sigma-1}\right) \tag{16}$$

Skilled sector

$$\omega_t\hat{Y}_t=\left(\frac{r_t^{k}}{\alpha}\right)^{\alpha}\left(\frac{\hat{\hat{w}}_t^{s}}{1-\alpha}\right)^{1-\alpha} \tag{17}$$

$$\left(\hat{K}_t^{s}\right)^{\alpha}\left(N_t^{s}\right)^{1-\alpha}=M_t f+\left(M_t-M_t^{x}\right)\left(\frac{1}{a}\frac{\chi_t}{1-\chi_t}\right)+\frac{M_t^{e}}{\chi_t^{e}}\left(f^{e}+\frac{1}{a^{e}}\frac{\chi_t^{e}}{1-\chi_t^{e}}\right) \tag{18}$$

$$\frac{r_t^{k}}{\hat{\hat{w}}_t^{s}}=\frac{\alpha}{1-\alpha}\frac{N_t^{s}}{\hat{K}_t^{s}} \tag{19}$$

Unskilled sector

$$\hat{Y}_t=Z_t M_t^{\frac{1}{\sigma-1}}\left(\hat{K}_t^{p}\right)^{\alpha}\left(N_t^{p}\right)^{1-\alpha} \tag{20}$$

$$\hat{\hat{w}}_t^{u}=(1-\alpha)\frac{\sigma-1}{\sigma}Z_t M_t^{\frac{1}{\sigma-1}}\left(\hat{K}_t^{p}\right)^{\alpha}\left(N_t^{p}\right)^{-\alpha}/\left(1+\zeta r_t^{l}\right) \tag{21}$$

$$r_t^{k}=\alpha\frac{\sigma-1}{\sigma}Z_t M_t^{\frac{1}{\sigma-1}}\left(\hat{K}_t^{p}\right)^{\alpha-1}\left(N_t^{p}\right)^{1-\alpha}/\left(1+\zeta r_t^{l}\right) \tag{22}$$

Households

$$1=\mathrm{E}_t\left[\beta\left(g_t^{Q}\cdot\hat{C}_{t+1}/\hat{C}_t\right)^{-\theta}\left(1+r_{t+1}^{d}\right)\right] \tag{23}$$

$$\Lambda_{t,t+1}=\mathrm{E}_t\left[\left(g_t^{Q}\cdot\hat{C}_{t+1}/\hat{C}_t\right)^{-\theta}\right] \tag{24}$$

Financial system

$$r_t^{l}=sp_t+r_t^{d} \tag{25}$$

$$r_t^{l}=r_t^{k}-d\mathrm{p} \tag{26}$$

Frictional labour markets (notation $w_t^* \equiv w_t(r)$)

$$m_t^u = \sigma_m \left(u_t^u\right)^{\psi} \left(v_t^u\right)^{1-\psi} \tag{27}$$

$$m_t^s = \sigma_m \left(u_t^s\right)^{\psi} \left(v_t^s\right)^{1-\psi} \tag{28}$$

$$n_{t+1}^u = \left(\rho^u + x_t^u\right) n_t^u \tag{29}$$

$$n_{t+1}^s = \left(\rho^s + x_t^s\right) n_t^s \tag{30}$$

$$u_t^u = 1 - n_t^u \tag{31}$$

$$u_t^s = 1 - n_t^s \tag{32}$$

$$q_t^u = \frac{m_t^u}{v_t^u} \tag{33}$$

$$q_t^s = \frac{m_t^s}{v_t^s} \tag{34}$$

$$p_t^u = \frac{m_t^u}{u_t^u} \tag{35}$$

$$p_t^s = \frac{m_t^s}{u_t^s} \tag{36}$$

$$x_t^u = \frac{q_t^u v_t^u}{n_t^u} \tag{37}$$

$$x_t^s = \frac{q_t^s v_t^s}{n_t^s} \tag{38}$$

$$\kappa^u x_t^u = \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\widehat{\widehat{w}}_{t+1}^u - \widehat{w}_t^u + \frac{\kappa^u}{2}\left(x_{t+1}^u\right)^2 + \rho^u \kappa^u x_{t+1}^u\right)\right] \tag{39}$$

$$\kappa^s x_t^s = \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\widehat{\widehat{w}}_{t+1}^s - \widehat{w}_t^s + \frac{\kappa^s}{2}\left(x_{t+1}^s\right)^2 + \rho^s \kappa^s x_{t+1}^s\right)\right] \tag{40}$$

$$\kappa^u x_t^{u^*} = \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\widehat{\widehat{w}}_{t+1}^u - \widehat{w}_t^{u^*} + \frac{\kappa^u}{2}\left(x_{t+1}^{u^*}\right)^2 + \rho^u \kappa^u x_{t+1}^{u^*}\right)\right] \tag{41}$$

$$\kappa^s x_t^{s^*} = \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\widehat{\widehat{w}}_{t+1}^s - \widehat{w}_t^{s^*} + \frac{\kappa^s}{2}\left(x_{t+1}^{s^*}\right)^2 + \rho^s \kappa^s x_{t+1}^{s^*}\right)\right] \tag{42}$$

$$\Delta_t^u = 1 + \rho^u \lambda \mathrm{E}_t\left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^u\right] \tag{43}$$

$$\Delta_t^s = 1 + \rho^s \lambda \mathrm{E}_t\left[\Lambda_{t,t+1} g_t^Q \Delta_{t+1}^s\right] \tag{44}$$

$$\Delta_t^u \widehat{w}_t^{u^*} = \widehat{w}_t^{uo} + \rho^u \lambda \mathrm{E}_t\left[\Lambda_{t,t+1} \Delta_{t+1}^u \widehat{w}_{t+1}^{u^*}\right] \tag{45}$$

$$\Delta_t^s \widehat{w}_t^{s^*} = \widehat{w}_t^{so} + \rho^s \lambda \mathrm{E}_t\left[\Lambda_{t,t+1} \Delta_{t+1}^s \widehat{w}_{t+1}^{s^*}\right] \tag{46}$$

$$\widehat{w}_t^{uf} = \psi\left(\widehat{\widehat{w}}_t^u + \frac{\kappa^u}{2}\left(x_t^u\right)^2 + p_t^u\kappa^u x_t^u\right) + \left(1-\psi\right)b_t^u \tag{47}$$

$$\widehat{w}_t^{sf} = \psi\left(\widehat{\widehat{w}}_t^s + \frac{\kappa^s}{2}\left(x_t^s\right)^2 + p_t^s\kappa^s x_t^s\right) + \left(1-\psi\right)b_t^s \tag{48}$$

$$\widehat{w}_t^{uo} = \widehat{w}_t^{uf} + \psi\left(\frac{\kappa^u}{2}\left(\left(x_t^{u*}\right)^2 - \left(x_t^u\right)^2\right) + p_t^u\kappa^u\left(x_t^{u*} - x_t^u\right)\right)$$

$$+\left(1-\psi\right)p_t^u \mathrm{E}_t\left[\Lambda_{t,t+1}\lambda\Delta_{t+1}^u g_t^Q\left(\widehat{w}_t^u - \widehat{w}_t^{u*}\right)\right] \tag{49}$$

$$\widehat{w}_t^{so} = \widehat{w}_t^{sf} + \psi\left(\frac{\kappa^s}{2}\left(\left(x_t^{s*}\right)^2 - \left(x_t^s\right)^2\right) + p_t^s\kappa^s\left(x_t^{s*} - x_t^s\right)\right)$$

$$+\left(1-\psi\right)p_t^s \mathrm{E}_t\left[\Lambda_{t,t+1}\lambda\Delta_{t+1}^s g_t^Q\left(\widehat{w}_t^s - \widehat{w}_t^{s*}\right)\right] \tag{50}$$

$$\widehat{w}_t^u = \lambda\widehat{w}_{t-1}^u + \left(1-\lambda\right)\widehat{w}_t^{u*} \tag{51}$$

$$\widehat{w}_t^s = \lambda\widehat{w}_{t-1}^s + \left(1-\lambda\right)\widehat{w}_t^{s*} \tag{52}$$

$$\widehat{b}_t^u = 0.4\widehat{w}_{ss}^u \tag{53}$$

$$\widehat{b}_t^s = 0.4\widehat{w}_{ss}^s \tag{54}$$

Market clearing

$$\hat{Y}_t = C_t + I_t + \kappa^u\left(x_t^u\right)^2 N_t^p + \kappa^s\left(x_t^s\right)^2 N_t^s \tag{55}$$

$$g_t^Q\hat{K}_{t+1} = \left(1-dp\right)\hat{K}_t + I_t \tag{56}$$

$$\hat{K}_t = \hat{K}_t^p + \hat{K}_t^s \tag{57}$$

$$N_t^p = \left(1-s\right)n_t^u \tag{58}$$

$$N_t^s = sn_t^s \tag{59}$$

Shocks

$$\log Z_t = \rho_Z \log Z_{t-1} + \varepsilon_{Z,t} \tag{60}$$

$$\log sp_t = \left(1-\rho_{\mathrm{sp}}\right)\log sp_{ss} + \rho_{\mathrm{sp}}\log sp_{t-1} + \varepsilon_{\mathrm{sp},t} \tag{61}$$

Welfare

$$U_t = \frac{\left(C_t Q_t\right)^{1-\theta}}{1-\theta} + \beta \mathrm{E}_t\left[U_{t+1}\right] \tag{62}$$

Additional derivations

Solutions of cost minimisation problems

Intermediate goods production sector

$$\min \quad tc_t^p(i) = \left(1+\zeta r_t^l\right)\left(\tilde{w}_t^u n_t^p(i) + r_t k_t^p(i)\right)$$

$$\text{subject to} \quad y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i) n_t^p(i)\right]^{1-\alpha}$$

FOCs

$$n_t(i) \quad : \quad \left(1+\zeta r_t^l\right)\tilde{w}_t^u = \lambda^p \left(1-\alpha\right) Z_t k_t^p(i)^\alpha q_t(i)^{1-\alpha} n_t^p(i)^{-\alpha}$$

$$k_t(i) \quad : \quad \left(1+\zeta r_t^l\right) r_t = \lambda^p \alpha Z_t k_t^p(i)^{\alpha-1} q_t(i)^{1-\alpha} n_t^p(i)^{1-\alpha}$$

Divide

$$\frac{\tilde{w}_t^u}{r_t} = \frac{1-\alpha}{\alpha}\frac{k_t^p(i)}{n_t^p(i)}$$

$$k_t^p(i) = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t^u}{r_t} n_t^p(i)$$

$$n_t^p(i) = \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u} k_t^p(i)$$

Production function

$$y_t(i) = Z_t k_t^p(i)^\alpha \left[q_t(i) n_t^p(i)\right]^{1-\alpha} = Z_t k_t^p(i)^\alpha \left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u} k_t^p(i)\right]^{1-\alpha}$$

$$= Z_t k_t^p(i)\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}$$

$$k_t^p(i) = \frac{y_t(i)}{Z_t}\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{\alpha-1}$$

Total cost

$$tc_t^p(i) = \left(1+\zeta r_t^l\right)\left(\tilde{w}_t^u n_t^p(i) + r_t k_t^p(i)\right) = \left(1+\zeta r_t^l\right)\left(\tilde{w}_t^u \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u} k_t^p(i) + r_t k_t^p(i)\right)$$

$$= \left(1+\zeta r_t^l\right)\left(\frac{1-\alpha}{\alpha}+1\right) r_t k_t^p(i) = \left(1+\zeta r_t^l\right)\frac{r_t}{\alpha} k_t^p(i)$$

$$= \left(1+\zeta r_t^l\right)\frac{r_t}{\alpha}\frac{y_t(i)}{Z_t}\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{\alpha-1} = \left(1+\zeta r_t^l\right)\frac{y_t(i)}{Z_t}\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}$$

Real marginal cost

$$mc_t^p(i) = \frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}$$

Research and development sector

$$\min \quad tc_t^x(i) = \left(1+\zeta r_t^l\right)\left(\tilde{w}_t^s n_t^x(i) + r_t k_t^x(i)\right)$$

$$\text{subject to} \quad x_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t \phi_t(i)}$$

FOCs

$$n_t^x(i) \quad : \quad \left(1+\zeta r_t^l\right)\tilde{w}_t^s = \lambda(1-\alpha)\frac{Z_t k_t^x(i)^\alpha Q_t^{1-\alpha} n_t^x(i)^{-\alpha}}{Q_t \phi_t(i)}$$

$$k_t^x(i) \quad : \quad \left(1+\zeta r_t^l\right) r_t = \lambda\alpha \frac{Z_t k_t^x(i)^{\alpha-1} Q_t^{1-\alpha} n_t^x(i)^{1-\alpha}}{Q_t \phi_t(i)}$$

Divide

$$\frac{\tilde{w}_t^s}{r_t} = \frac{1-\alpha}{\alpha}\frac{k_t^x(i)}{n_t^x(i)}$$

$$k_t^x(i) = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t^s}{r_t} n_t^x(i)$$

$$n_t^x(i) = \frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s} k_t^x(i)$$

R&D production function

$$x_t(i) = \frac{k_t^x(i)^\alpha \left[Q_t n_t^x(i)\right]^{1-\alpha}}{Q_t \phi_t(i)} = Q_t^{-\alpha} k_t^x(i)\left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{1-\alpha} / \phi_t(i)$$

$$k_t^x(i) = x_t(i) Q_t^\alpha \left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{\alpha-1} \phi_t(i)$$

Total cost

$$tc_t^x(i) = \left(1+\zeta r_t^l\right)\frac{r_t}{\alpha} k_t^x(i) = \left(1+\zeta r_t^l\right)\frac{r_t}{\alpha} x_t(i) Q_t^\alpha \left(\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^s}\right)^{\alpha-1} \phi_t(i)$$

$$= \left(1+\zeta r_t^l\right) x_t(i) Q_t^\alpha \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha} \phi_t(i)$$

Real marginal cost

$$mc_t^x(i) = \left(1+\zeta r_t^l\right) Q_t^\alpha \left(\frac{r_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t^s}{1-\alpha}\right)^{1-\alpha} \phi_t(i) \equiv \overline{m}c_t^x \phi_t(i)$$

Total cost as function of desired innovative success probability

$$\chi_t(i) = \frac{a x_t(i)}{1 + a x_t(i)}$$

$$x_t(i) = \frac{1}{a}\frac{\chi_t(i)}{1-\chi_t(i)}$$

$$tc_t^x(i) = \left(1+\zeta r_t^l\right)\frac{\overline{m}c_t^x}{a}\frac{\chi_t(i)}{1-\chi_t(i)}\phi_t(i)$$

Aggregate production function

Relative inputs

$$\frac{y_t(i)}{y_t(j)} = \frac{Y_t p_t(i)^{-\sigma}}{Y_t p_t(j)^{-\sigma}} = \left[\frac{\frac{\sigma}{\sigma-1}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}}{\frac{\sigma}{\sigma-1}\frac{\left(1+\zeta r_t^l\right)}{Z_t}\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t^u / q_t(j)}{1-\alpha}\right)^{1-\alpha}}\right]^{-\sigma} = \left(\frac{q_t(i)^{\alpha-1}}{q_t(j)^{\alpha-1}}\right)^{-\sigma} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}}\right)^{\sigma}$$

$$\frac{y_t(i)}{y_t(j)} = \frac{Z_t k_t^p(i)\left[q_t(i)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}}{Z_t k_t^p(j)\left[q_t(j)\frac{1-\alpha}{\alpha}\frac{r_t}{\tilde{w}_t^u}\right]^{1-\alpha}}$$

$$\frac{k_t^p(i) q_t(i)^{1-\alpha}}{k_t^p(j) q_t(j)^{1-\alpha}} = \left(\frac{q_t(i)^{1-\alpha}}{q_t(j)^{1-\alpha}}\right)^{\sigma}$$

$$\frac{k_t^p(i)}{k_t^p(j)} = \left(\frac{q_t(i)}{q_t(j)}\right)^{(1-\alpha)(\sigma-1)}$$

$$k_t^p(i) = \left(\frac{q_t(i)}{q_t(j)}\right)^{(1-\alpha)(\sigma-1)} k_t^p(j)$$

$$k_t^p(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \overline{k}_t^p$$

$$n_t^p(i) = \left(\frac{q_t(i)}{Q_t}\right)^{(1-\alpha)(\sigma-1)} \overline{n}_t^p$$

where $\overline{k}_t^p \equiv K_t^p / M_t$ and $\overline{n}_t^p \equiv N_t^p / M_t$.

Final goods output

$$Y_t = \left[\int_0^{M_t} y_t(i)^{\frac{\sigma-1}{\sigma}} \mathrm{d}i\right]^{\frac{\sigma}{\sigma-1}} = \left[M_t \int_0^{\infty} y_t(q)^{\frac{\sigma-1}{\sigma}} \mu_t(q)\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}}$$

$$= M_t^{\frac{\sigma}{\sigma-1}}\left[\int_0^{\infty}\left[Z_t k_t^p(q)^{\alpha} q^{1-\alpha} n_t^p(q)^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}} \mu_t(q)\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}}$$

$$= M_t^{\frac{\sigma}{\sigma-1}} Z_t\left[\int_0^{\infty}\left[\left(\frac{q}{Q_t}\right)^{(1-\alpha)(\sigma-1)}\left(\overline{k}_t^p\right)^{\alpha}\left(\overline{n}_t^p\right)^{1-\alpha} q^{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}} \mu_t(q)\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}}$$

$$= M_t^{\frac{\sigma}{\sigma-1}} Z_t\left(\overline{k}_t^p\right)^{\alpha}\left(\overline{n}_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left[\int_0^{\infty}\left[\left(q^{1-\alpha}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}} \mu_t(q)\mathrm{d}q\right]^{\frac{\sigma}{\sigma-1}}$$

$$= M_t^{\frac{\sigma}{\sigma-1}-1} Z_t\left(K_t^p\right)^{\alpha}\left(N_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)}\left[\left[\int_0^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} \mu_t(q)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{\sigma}$$

$$= M_t^{\frac{1}{\sigma-1}} Z_t \left(K_t^p\right)^{\alpha} \left(N_t^p\right)^{1-\alpha} Q_t^{(1-\alpha)(1-\sigma)} \left(Q_t^{1-\alpha}\right)^{\sigma}$$

$$= M_t^{\frac{1}{\sigma-1}} Z_t \left(K_t^p\right)^{\alpha} \left(Q_t N_t^p\right)^{1-\alpha}$$

Real profit function

Real operating profit

$$\pi_t^o(i) = p_t(i) y_t(i) - mc_t^p(i) y_t(i) - f_t = p_t(i) y_t(i) - p_t(i) \frac{\sigma-1}{\sigma} y_t(i) - f_t$$

$$= \left(1 - \frac{\sigma-1}{\sigma}\right) Y_t p_t(i)^{1-\sigma} - f_t = \frac{1}{\sigma} Y_t \left[\frac{\sigma}{\sigma-1} \frac{\left(1+\zeta r_t^l\right)}{Z_t} \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t$$

$$= \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} Y_t Z_t^{\sigma-1} \left[\left(1+\zeta r_t^l\right) \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t^u / q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t$$

Price index (where $R_t \equiv P_t r_t$ and $W_t^u \equiv P_t \tilde{w}_t^u$)

$$P_t = \left[\int_0^{M_t} P_t(i)^{1-\sigma} \mathrm{d}i\right]^{\frac{1}{1-\sigma}} = \left[M_t \int_0^{\infty} P_t(q)^{1-\sigma} \mu_t(q) \mathrm{d}q\right]^{\frac{1}{1-\sigma}}$$

$$= M_t^{\frac{1}{1-\sigma}} \left[\int_0^{\infty} \left[\frac{\sigma}{\sigma-1} \frac{\left(1+\zeta r_t^l\right)}{Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t^u / q}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} \mu_t(q) \mathrm{d}q\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{\left(1+\zeta r_t^l\right)}{Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left[\int_0^{\infty} \left(q^{\alpha-1}\right)^{1-\sigma} \mu_t(q) \mathrm{d}q\right]^{\frac{1}{1-\sigma}}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{\left(1+\zeta r_t^l\right)}{Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left[\left[\int_0^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} \mu_t(q) \mathrm{d}q\right]^{\frac{1}{\sigma-1}}\right]^{-1}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{\left(1+\zeta r_t^l\right)}{Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t^u}{1-\alpha}\right)^{1-\alpha} \left(Q_t^{1-\alpha}\right)^{-1}$$

$$= \frac{\sigma}{\sigma-1} M_t^{\frac{1}{1-\sigma}} \frac{\left(1+\zeta r_t^l\right)}{Z_t} \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t^u / Q_t}{1-\alpha}\right)^{1-\alpha}$$

Real input cost index

$$\left(1+\zeta r_t^l\right) \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t^u / Q_t}{1-\alpha}\right)^{1-\alpha} = \frac{\sigma-1}{\sigma} P_t M_t^{\frac{1}{\sigma-1}} Z_t$$

$$\left(1+\zeta r_t^l\right) \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t^u}{1-\alpha}\right)^{1-\alpha} = \frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha}$$

Real operating profit

$$\pi_t^o(i) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} Y_t Z_t^{\sigma-1} \left[\left(1+\zeta r_t^l\right) \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t / q_t(i)}{1-\alpha}\right)^{1-\alpha}\right]^{1-\sigma} - f_t$$

$$= \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} Y_t Z_t^{\sigma-1} \left[\frac{\sigma-1}{\sigma} M_t^{\frac{1}{\sigma-1}} Z_t Q_t^{1-\alpha} q_t(i)^{\alpha-1} \right]^{1-\sigma} - f_t$$

$$= \frac{Y_t}{\sigma M_t} \left[\left(\frac{q_t(i)}{Q_t} \right)^{1-\alpha} \right]^{\sigma-1} - f_t = \frac{Y_t}{\sigma M_t} \phi_t(i) - f_t$$

Real profit

$$\pi_t(i) = \pi_t^o(i) - \left(1+\zeta r_t^l\right) \frac{\overline{m}c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \phi_t(i)$$

$$= \left(\frac{Y_t}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\overline{m}c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - f_t$$

$$= \left(\frac{Y_t}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\overline{m}c_t^x}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - \left(1+\zeta r_t^l\right) \overline{m}c_t^x f$$

$$= Y_t \left[\left(\frac{1}{\sigma M_t} - \left(1+\zeta r_t^l\right) \frac{\omega_t}{a} \frac{\chi_t(i)}{1-\chi_t(i)} \right) \phi_t(i) - \left(1+\zeta r_t^l\right) \omega_t f \right]$$

Evolution of aggregate quality index

Following Melitz [2003], I consider the current period distribution of quality levels $\mu_t(q)$ to be a truncated part of the underlying distribution $g_t(q)$, so that:

$$\mu_t(q) = \begin{cases} 1/\left[1-G_t\left(q_{t-1}^*\right)\right] g_t(q) & \text{if } q \geq q_{t-1}^* \\ 0 & \text{otherwise} \end{cases}$$

where $q_t^* = \left(\phi_t^*\right)^{1/[(1-\alpha)(\sigma-1)]} Q_t$.
Aggregate quality index at the end of period t:

$$Q_t^{1-\alpha} = \left[\int_0^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} \mu_t(q) \mathrm{d}q \right]^{\frac{1}{\sigma-1}} = \left[\frac{1}{1-G_t\left(q_{t-1}^*\right)} \int_{q_{t-1}^*}^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q) \mathrm{d}q \right]^{\frac{1}{\sigma-1}}$$

Aggregate quality level after exits and innovation resolution but before entry:

$$Q_t^* = \left\{ \frac{1}{1-G_t\left(q_t^*\right)} \left[\left(1-\chi_t\right) \int_{q_t^*}^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q) \mathrm{d}q + \chi_t \int_{q_t^*}^{\infty} \left(\iota^{\frac{1}{(1-\alpha)(\sigma-1)}} q \right)^{(1-\alpha)(\sigma-1)} g_t(q) \mathrm{d}q \right] \right\}^{\frac{1}{\sigma-1}}$$

$$= \left[\left(1-\chi_t+\chi_t \iota\right) \frac{1}{1-G_t\left(q_t^*\right)} \int_{q_t^*}^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q) \mathrm{d}q \right]^{\frac{1}{\sigma-1}}$$

Aggregate quality index in $t+1$ after entry:

$$Q_{t+1} = \left\{ \frac{1-\chi_t+\chi_t \iota}{1-G_t\left(q_t^*\right)} \left[\left(1-\frac{M_t^e}{M_{t+1}}\right) \int_{q_t^*}^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q) \mathrm{d}q + \frac{M_t^e}{M_{t+1}} \int_{q_t^*}^{\infty} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\frac{1}{(1-\alpha)(\sigma-1)}} q \right)^{(1-\alpha)(\sigma-1)} g_t(q) \mathrm{d}q \right] \right\}^{\frac{1}{\sigma-1}}$$

$$= \left[\left(1-\chi_t+\chi_t \iota\right) \left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}} \frac{\sigma}{\sigma-1}\right) \frac{1}{1-G_t\left(q_t^*\right)} \int_{q_t^*}^{\infty} \left(q^{1-\alpha}\right)^{\sigma-1} g_t(q) \mathrm{d}q \right]^{\frac{1}{\sigma-1}}$$

Transformed aggregate growth rate η_t:

$$\eta_t = \left(\frac{Q_{t+1}}{Q_t}\right)^{(1-\alpha)(\sigma-1)} = \left\{ \frac{\left[\left(1-\chi_t+\chi_t \iota\right)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)\frac{1}{1-G_t\left(q_t^*\right)}\int_{q_t^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\mathrm{d}q \right]^{\frac{1}{\sigma-1}}}{\left[\frac{1}{1-G_t\left(q_{t-1}^*\right)}\int_{q_{t-1}^*}^{\infty}\left(q^{1-\alpha}\right)^{\sigma-1} g_t(q)\mathrm{d}q\right]^{\frac{1}{\sigma-1}}} \right\}^{\sigma-1}$$

$$\approx \left(1-\chi_t+\chi_t \iota\right)\left(1-\frac{M_t^e}{M_{t+1}}+\frac{M_t^e}{M_{t+1}}\frac{\sigma}{\sigma-1}\right)$$

where if the distribution is invariant with respect to the cut-off points q_{t-1}^* and q_t^* (as is the case with Pareto and other power-law distributions) then the above relationship holds with equality.

Wages

Denote by $W_t(j)$ the expected discounted sum of future wages received over the duration of the relationship with the employment agency:

$$W_t(j) = \Delta_t w_t(j) + (1-\lambda)\mathrm{E}_t \sum_{s=1}^{\infty} \left(\beta\rho\right)^s \Lambda_{t,t+s}\Delta_{t+s} w_{t+s}(r)$$

where the first part represents a contract that is not renegotiated and the wage is only indexed, while the second part represents future, renegotiated contracts at the same employment agency, and:

$$\Delta_t = \mathrm{E}_t \sum_{s=1}^{\infty} \left(\beta\rho\lambda\right)^s \Lambda_{t,t+s}\frac{Q_{t+s}}{Q_t}$$

The match surplus of workers employed by renegotiating agency r can then be rewritten as:

$$\begin{aligned} H_t(r) &= w_t(r) + \mathrm{E}_t\left[\Lambda_{t,t+1}\rho H_{t+1}(r)\right] - b_t - \mathrm{E}_t\left[\Lambda_{t,t+1} p_t H_{t+1}\right] \\ &= W_t(r) - \mathrm{E}_t \sum_{s=1}^{\infty}\left(\beta\rho\right)^s \Lambda_{t,t+s}\left(b_{t+s} + p_{t+s} H_{t+s+1}\right) \end{aligned}$$

Similarly, the match surplus of an employed worker from the point of view of the employment agency can be rewritten as:

$$\begin{aligned} J_t(r) &= \tilde{w}_t + \frac{\kappa}{2}x_t^2(r) + \rho\mathrm{E}_t\left[\beta\Lambda_{t,t+1} J_{t+1}(r)\right] - w_t(r) \\ &= \mathrm{E}_t \sum_{s=1}^{\infty}\left(\beta\rho\right)^s \Lambda_{t,t+s}\left(\tilde{w}_{t+s} + \frac{\kappa}{2}x_{t+s}^2(r)\right) - W_t(r) \end{aligned}$$

By substituting the above expressions in the surplus sharing equation one can obtain:

$$W_t(r) = \psi\mathrm{E}_t \sum_{s=0}^{\infty}\left(\beta\rho\right)^s \Lambda_{t,t+s}\left(\tilde{w}_{t+s} + \frac{\kappa}{2}x_{t+s}^2(r)\right) + \left(1-\psi\right)\mathrm{E}_t \sum_{s=1}^{\infty}\left(\beta\rho\right)^s \Lambda_{t,t+s}\left(b_{t+s} + p_{t+s} H_{t+s+1}\right)$$

or, after simplifying, in the following recursive form:

$$\Delta_t w_t(r) = \psi\left(\tilde{w}_t + \frac{\kappa}{2}x_t^2(r)\right) + \left(1-\psi\right)\left(b_t + p_t\mathrm{E}_t\left[\Lambda_{t,t+1}H_{t+1}\right]\right) + \rho\lambda\mathrm{E}_t\left[\Lambda_{t,t+1}\Delta_{t+1} w_{t+1}(r)\right]$$

where the first two terms comprise the target wage w_t^o, which in turn can be expressed in relation to the flexible contract wage:

$$
\begin{aligned}
w_t^o &= \psi\left(\tilde{w}_t + \frac{\kappa}{2}x_t^2(r)\right) + (1-\psi)\left(b_t + p_t \mathrm{E}_t\left[\Lambda_{t,t+1}H_{t+1}\right]\right) \\
&= w_t^f + \psi\left(\frac{\kappa}{2}\left(x_t^2(r) - x_t^2\right) - p_t\kappa x_t\right) + (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}H_{t+1}\right]
\end{aligned}
$$

Average vs conditional on renegotiation worker surplus

$$
H_t = H_t(r) + \Delta_t\left(w_t - w_t(r)\right)
$$

Therefore

$$
\begin{aligned}
&(1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}H_{t+1}\right] = \\
&= (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}\left[H_{t+1}(r) + \lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]\right] \\
&= (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}H_{t+1}(r)\right] + (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right] \\
&= \psi p_t \mathrm{E}_t\left[\Lambda_{t,t+1}J_{t+1}(r)\right] + (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right] \\
&= \psi p_t\kappa x_t(r) + (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]
\end{aligned}
$$

Resulting target wage

$$
w_t^o = w_t^f + \psi\left(\frac{\kappa}{2}\left(x_t^2(r) - x_t^2\right) + p_t\kappa\left(x_t(r) - x_t\right)\right) + (1-\psi)p_t \mathrm{E}_t\left[\Lambda_{t,t+1}\lambda\Delta_{t+1}\left(w_{t+1} - w_{t+1}(r)\right)\right]
$$

The above equation emphasises the presence of spill-overs of economy-wide wages on the bargaining wage. Intuitively, more intensive hiring by an agency requires higher bargained wages, which are also upwardly pressured by the future average wage.

Finally, let x_t denote the average hiring rate:

$$
x_t = \int_0^1 x_t(j)\frac{n_t(j)}{n_t}\mathrm{d}j
$$

Then the job creation condition can be used to express x_t as:

$$
\begin{aligned}
\kappa x_t &= \mathrm{E}_t\left[\Lambda_{t,t+1}\left(\tilde{w}_{t+1} - w_{t+1} + \frac{\kappa}{2}x_{t+1}^2 + \rho\kappa x_{t+1}\right)\right] \\
&+ \mathrm{E}_t\left[\Lambda_{t,t+1}\int_0^1\left(\frac{\kappa}{2}x_{t+1}^2(j) + \rho\kappa x_{t+1}(j) - w_{t+1}(j)\right)\frac{n_t(j)}{n_t}\mathrm{d}j - \left(\frac{\kappa}{2}x_{t+1}^2 + \rho\kappa x_{t+1} - w_{t+1}\right)\right]
\end{aligned}
$$

Note that along the balanced growth path the deviations of individual employment agencies' decisions from the average disappear and one can only take the first line of the above equation as a first-order approximation.